UNIT-I:

Fundamentals of Control Systems: Definition of System, Control System, Different examples of Control Systems, Classification of Control Systems, Open Loop and Closed loop Control System and their differences, Effects of feedback.

Representation of Control systems: Transfer function, Block diagram algebra, Reduction of Block diagrams, Signal flow graphs, Reduction of Signal flow graph using Mason's gain formula.

System

- System is a collection of components to produce desired objective
- System produces an output (also called response) for an input (called excitation).

Motor

Input- Electrical Energy (Voltage)
Output – Mechanical Energy (Rotation / Torque)

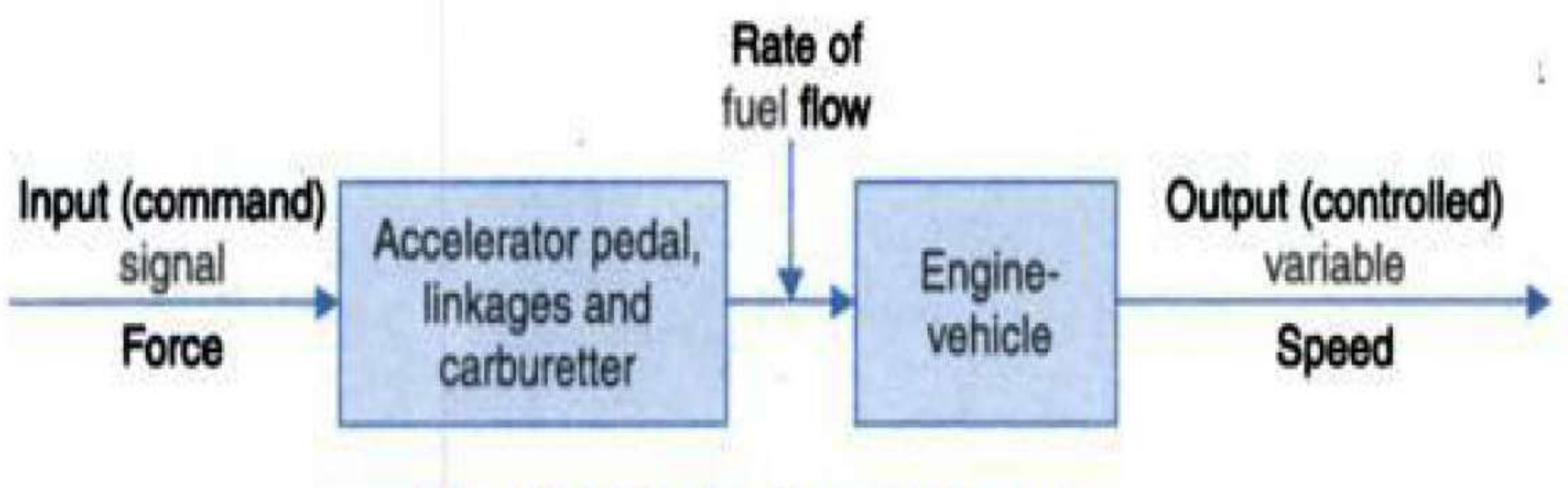
Car

Input- Acceleration
Output- Vehicle Displacement

Control System

Control System- A system in which any **quantity of interest** in a machine, mechanism or other equipment is maintained or **altered** in accordance with a desired manner

Driving System in a Car



Open Loop and Closed Loop Control Systems

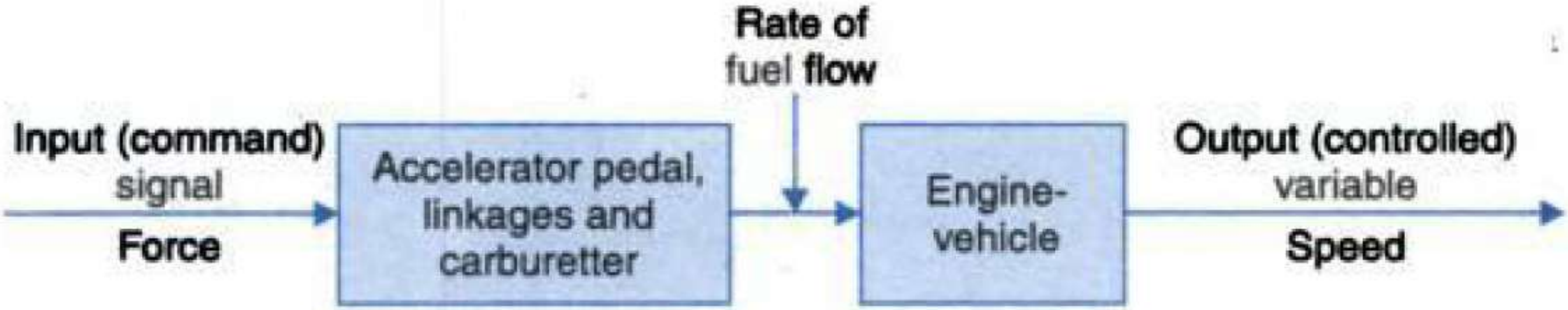
Open Loop Control System : A control System in which the controlling process is totally **independent on output of a system and depends only on input**

Closed Loop Control System : A control System in which the controlling process is **Depends on both input and output.**

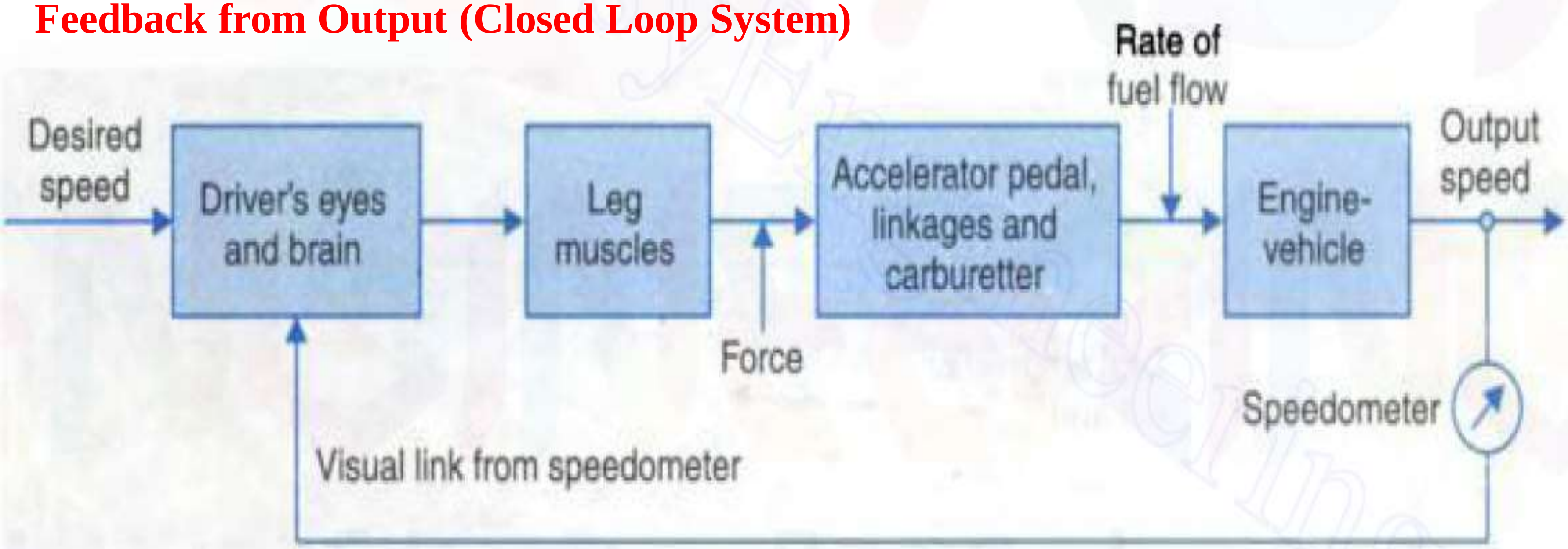
Examples of Open Loop and Closed Loop Control Systems

Driving System in a Car

No Feedback from Output (Open Loop System)

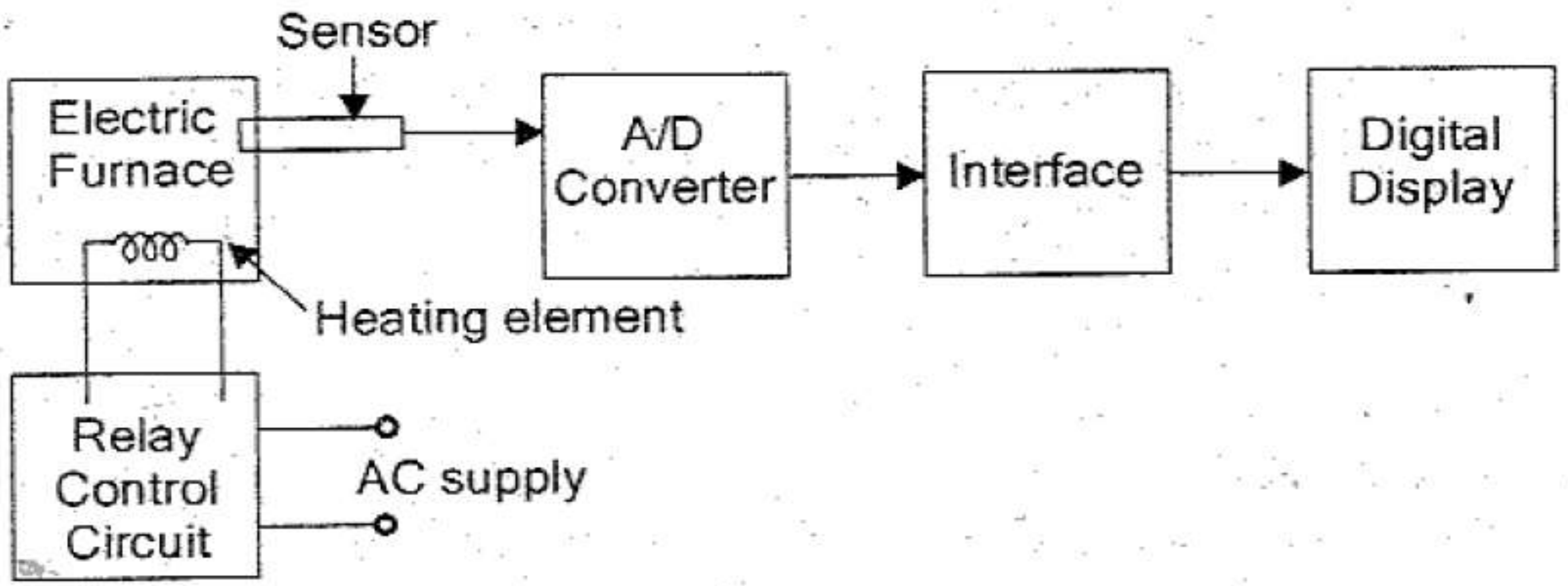


Feedback from Output (Closed Loop System)



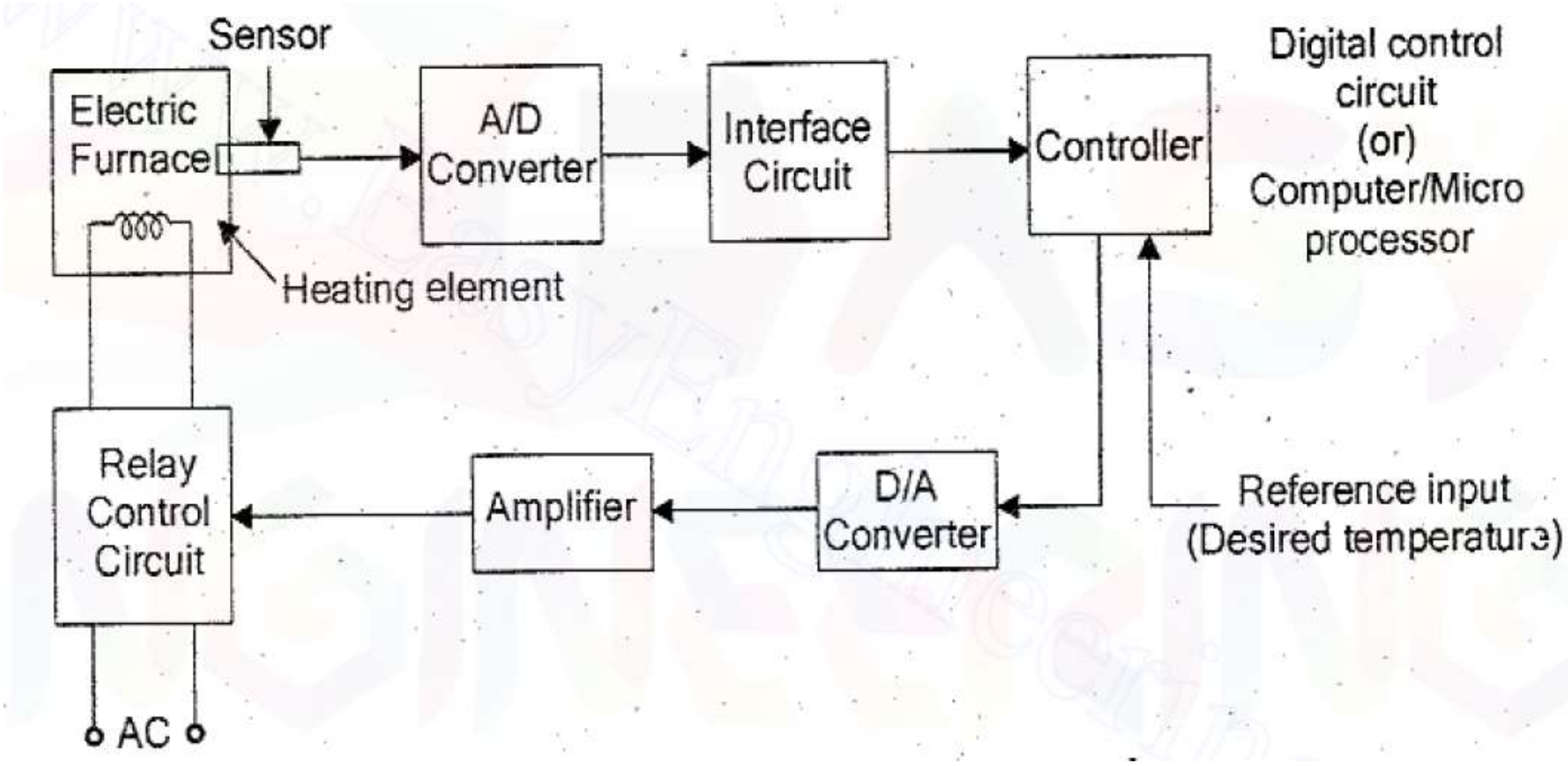
Temperature Control System

No Feedback from Output (Open Loop System)



Temperature Control System

Feedback from Output (Closed Loop System)



Examples of Open Loop Control Systems

1. **Bread Toaster**
   (Irrespective of bread is completely toast or not it will heat upto setup time)

2. **Electric Hand Drier**
   (Hand drier produce hot air upto some time, irrespective of hand is dried or not)

3. **Cloth Drier**
   (Machine stops working automatically after some time (20 mins), irrespective of the nature of the clothes, whether they are dry or not

4. **Automatic Cofee Maker**

5. **Traffi Light Controller**
   (Irrespective of Traffic , the lights Switch on and off for predefined time)

Examples of Closed Loop Control Systems

1. **Automatic Traffic light Control**
   (Traffic lights are automatically control based on the density of traffic)

2. **Air Conditioner (Temperature Control System)**
   (The temperature set by user is maintained irrespective of tempeature in room and number of people in the room)

3. **Voltage Regulator**
   (Irrespective of voltage fluctuation in home the output of regulator is maintained constant voltage)

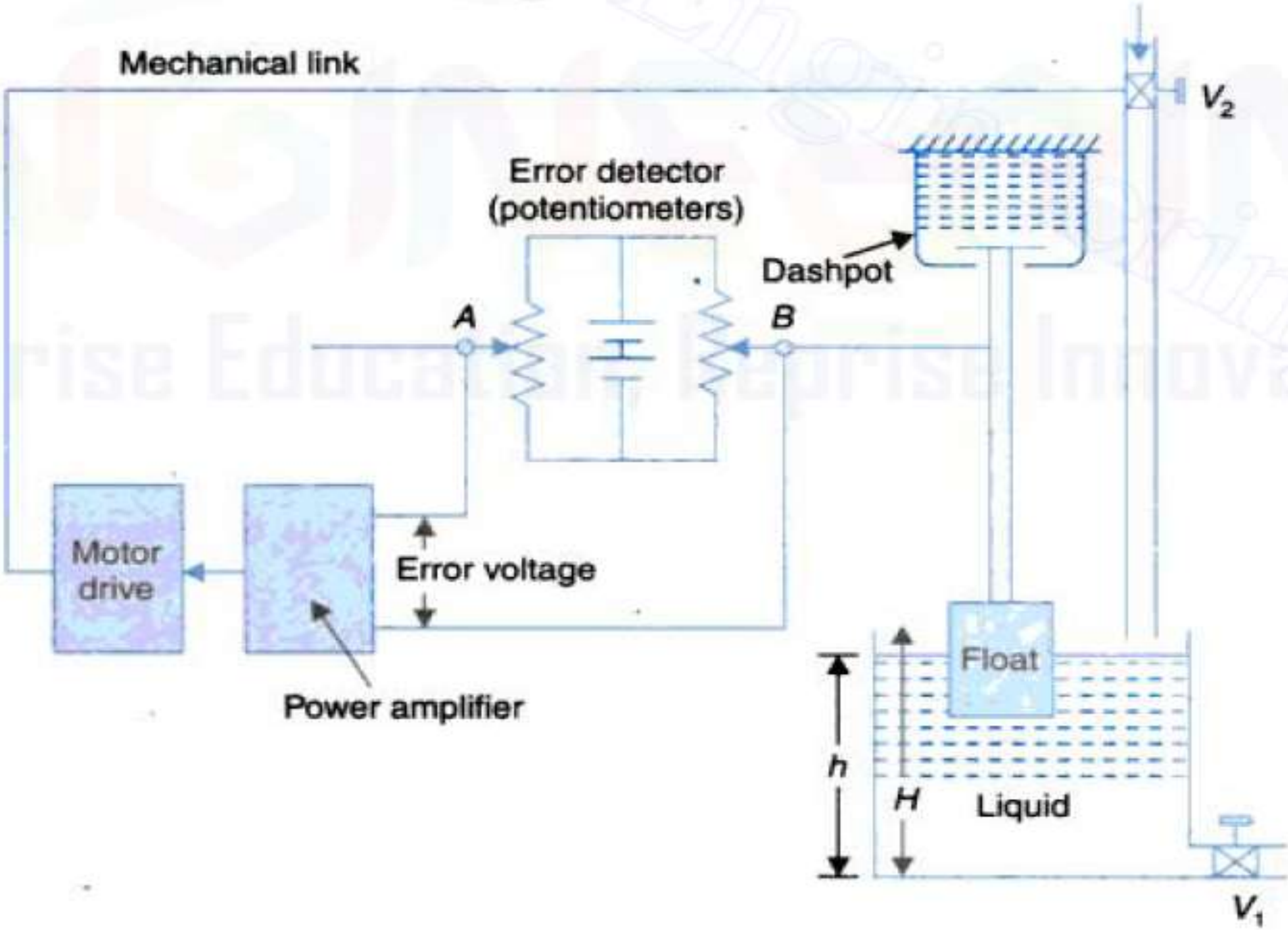
4. **Automatic Water Tank Level Control System**
   (Desired liquid level is maintained even though the output flow rate is varied)

5. **Rocket Autopilot System & Automatic Fligt Landing System**

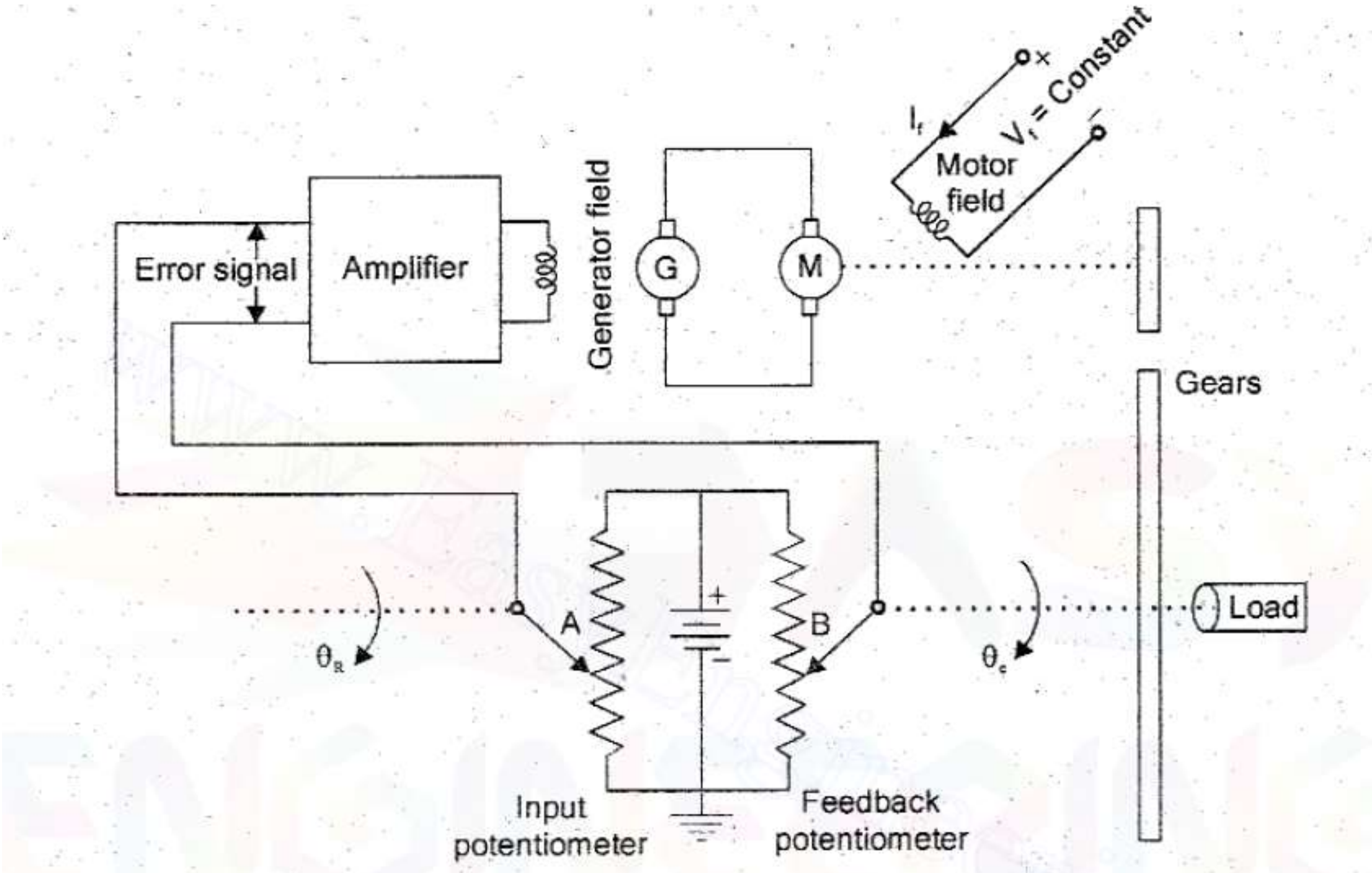
6. **Missile Launching System**
   (Irrespective of Traffic , the lights Switch on and off for predefined time)

7. **Human Body Temperature & Human Respiratory System**

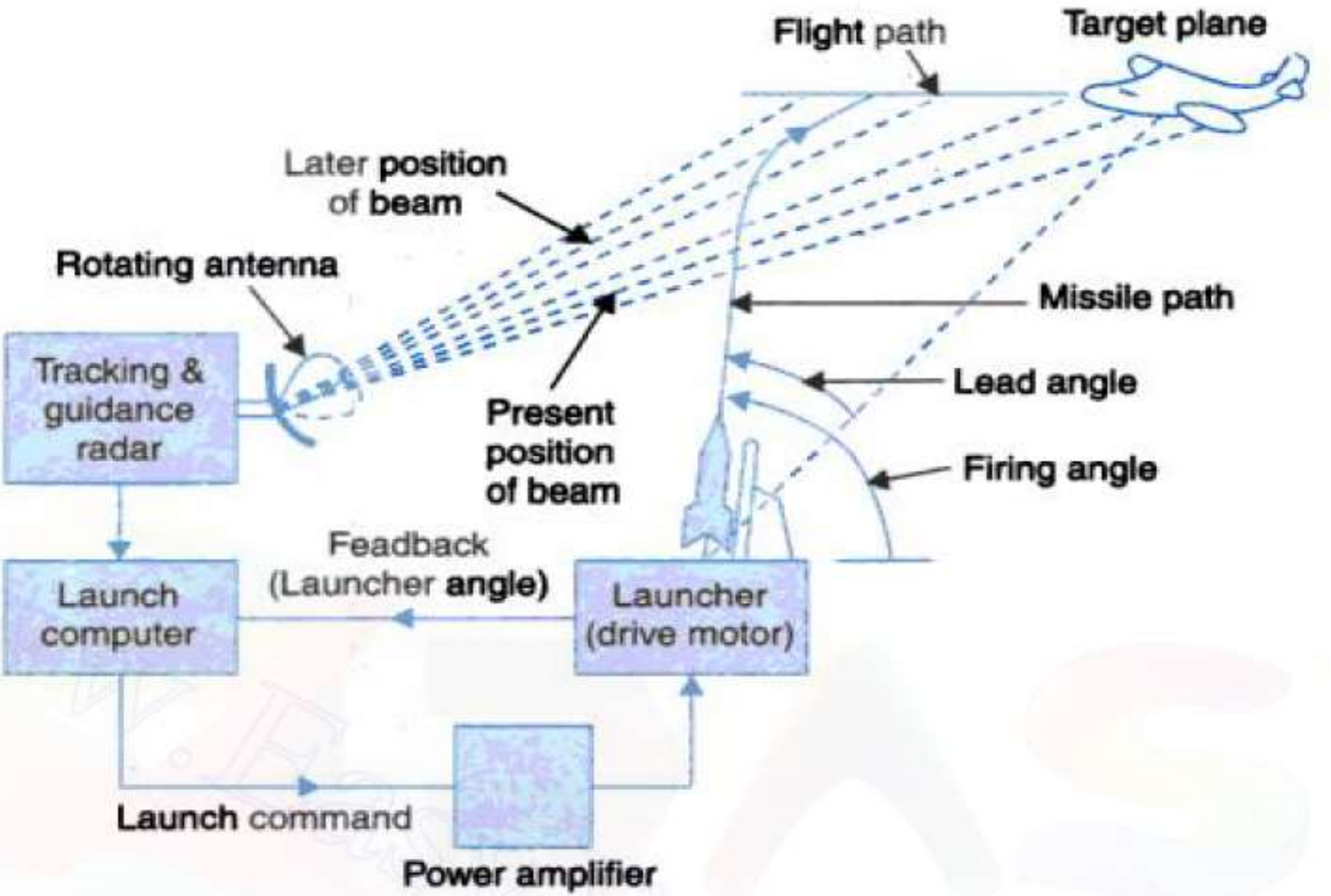
Water Level Control System



Position Control System



Missile Launching Control System



Classification of Control Systems

Classification of Control Systems

1. Open loop and Closed loop Control System
2. Linear and Non Linear Control System
3. Time Variant and Tme Invariant Control Systems
4. Discrete and Continuous Control Systems
5. Single input single output (SISO) and Multiple input and Multiple output (MIMO) control systems
6. Manual and Automatic Control Systems
7. Other types like position control systems, velocity control systems, Type -0, Type-1, Type-2 systems, Undamped systems, under damped systems, critically damped systems, Overdamped systems, First order control system, second order control system.

Open loop and Closed loop Control System

A system in which the controlling process depends on input only is called open loop control system

A system in which the controlling process depends on input and output is called Closed loop control system

Linear and Non Linear Control System

A system which satisfies superposition theorem (Additivity and Homogenity) is called Linear system

A system which does not satisfies superposition theorem (Additivity and Homogenity) is called Non- Linear system

Time Variant and Tme Invariant Control Systems

When the parameters of control system are stationary with respect to time it is called time-invariant system

When the parameters of system vary with respect to time is called time variant system

Discrete and Continuous Control Systems

If the controlling action is discrete in manner then it is called discrete control systems

If the controlling action is continuous in manner then it is called continuous control systems

Single input single output (SISO) and Multiple input and Multiple output (MIMO) control systems

If the control system has single input and single output, then it is called Single input and single output system

If the control system has more than one input and output, then it is called Multiple input and Multiple output system

Manual and Automatic Control Systems

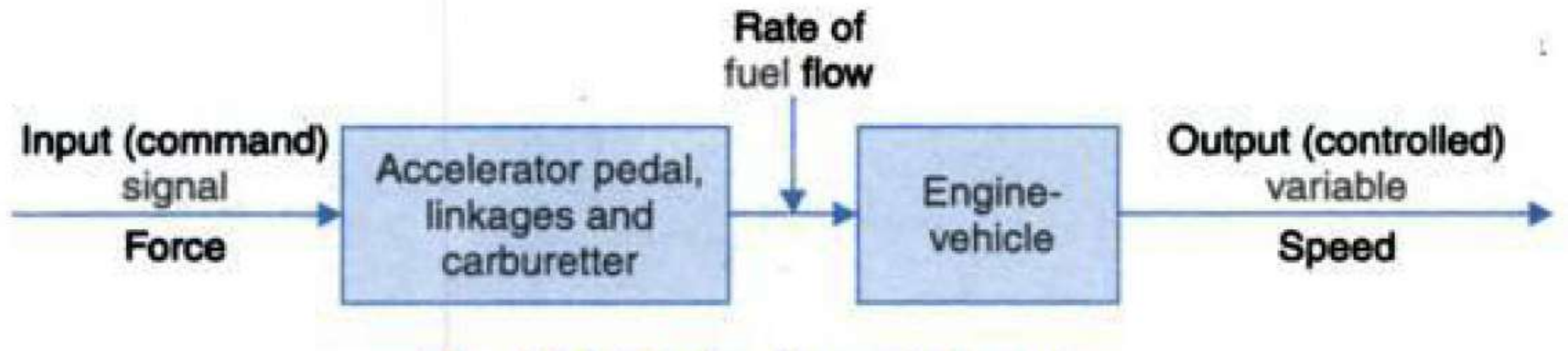
If the system is controlled manually it is caled manual control system

If the system is controlled automatically with out any human intervention is automatic control system

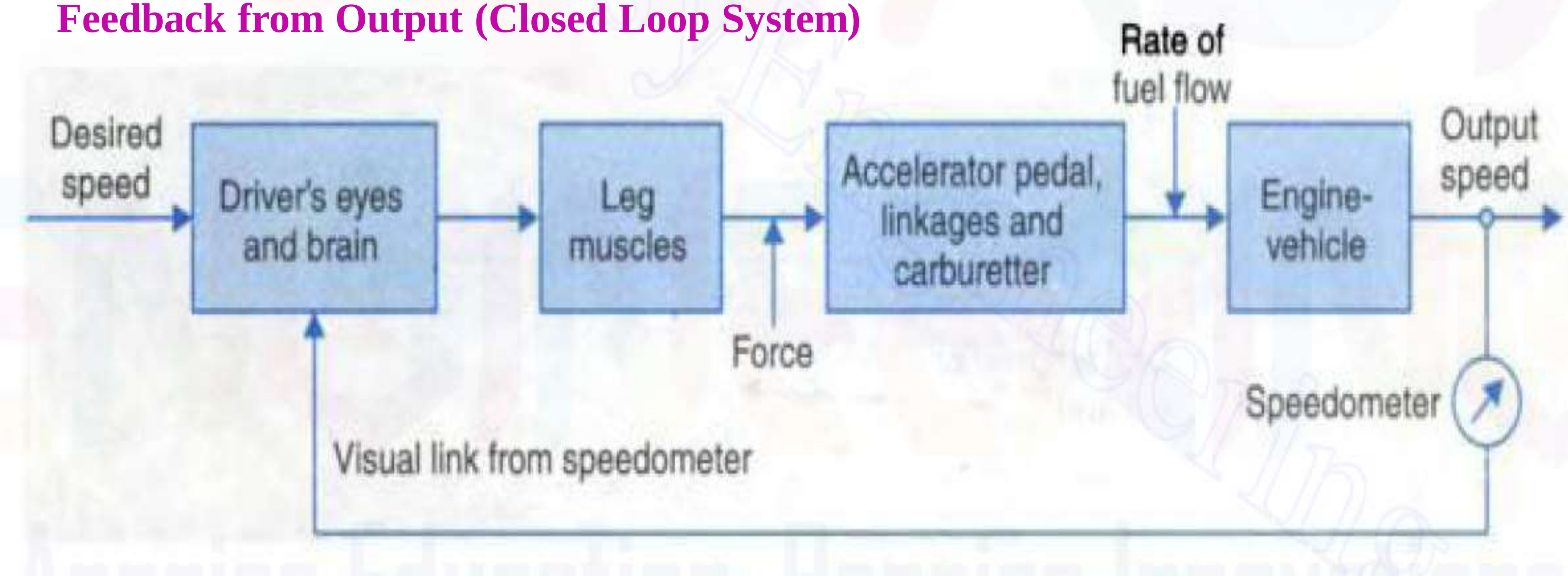
General Representation of Open Loop and Closed Loop Control Systems

Driving System in a Car

No Feedback from Output (Open Loop System)

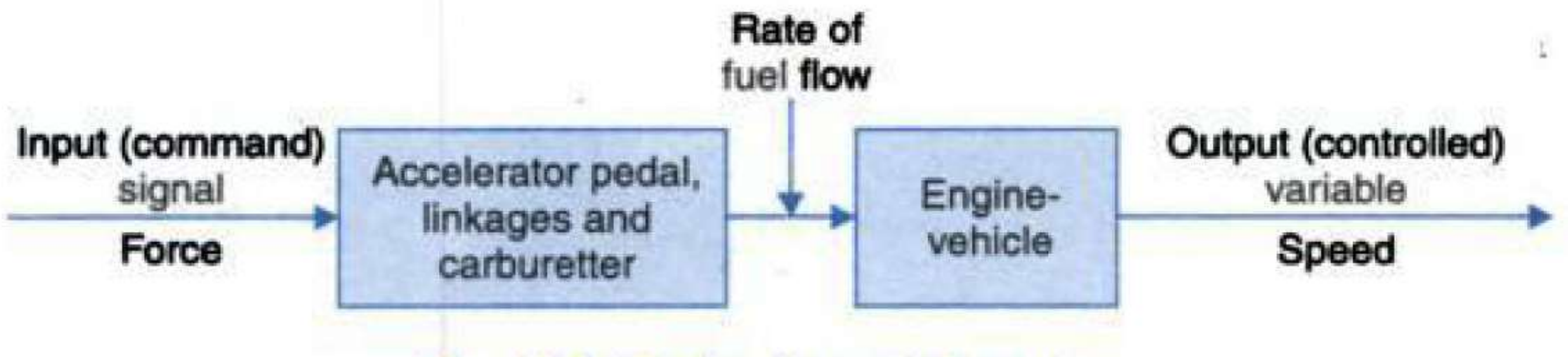


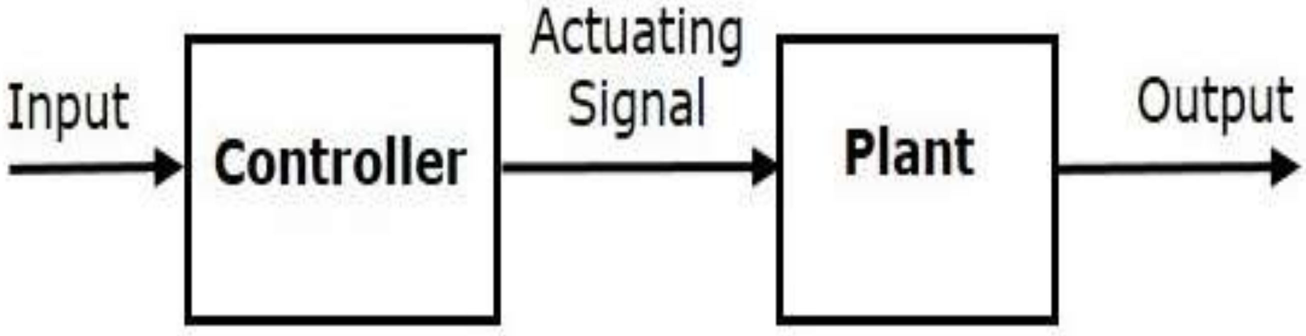
Feedback from Output (Closed Loop System)



General Block Diagram of Open Loop Control System

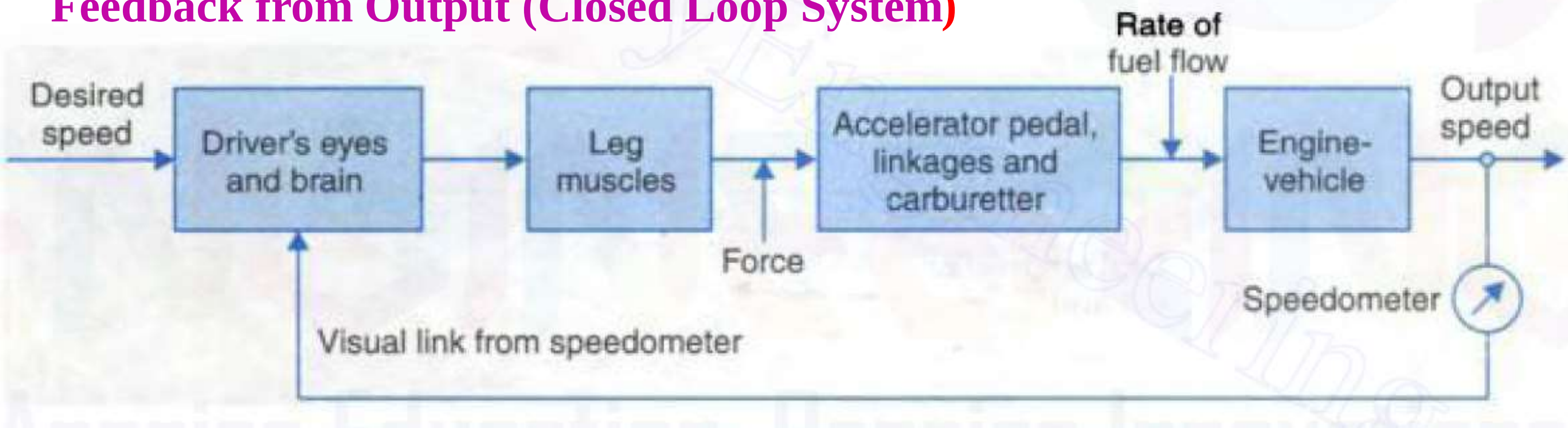
No Feedback from Output (Open Loop System)

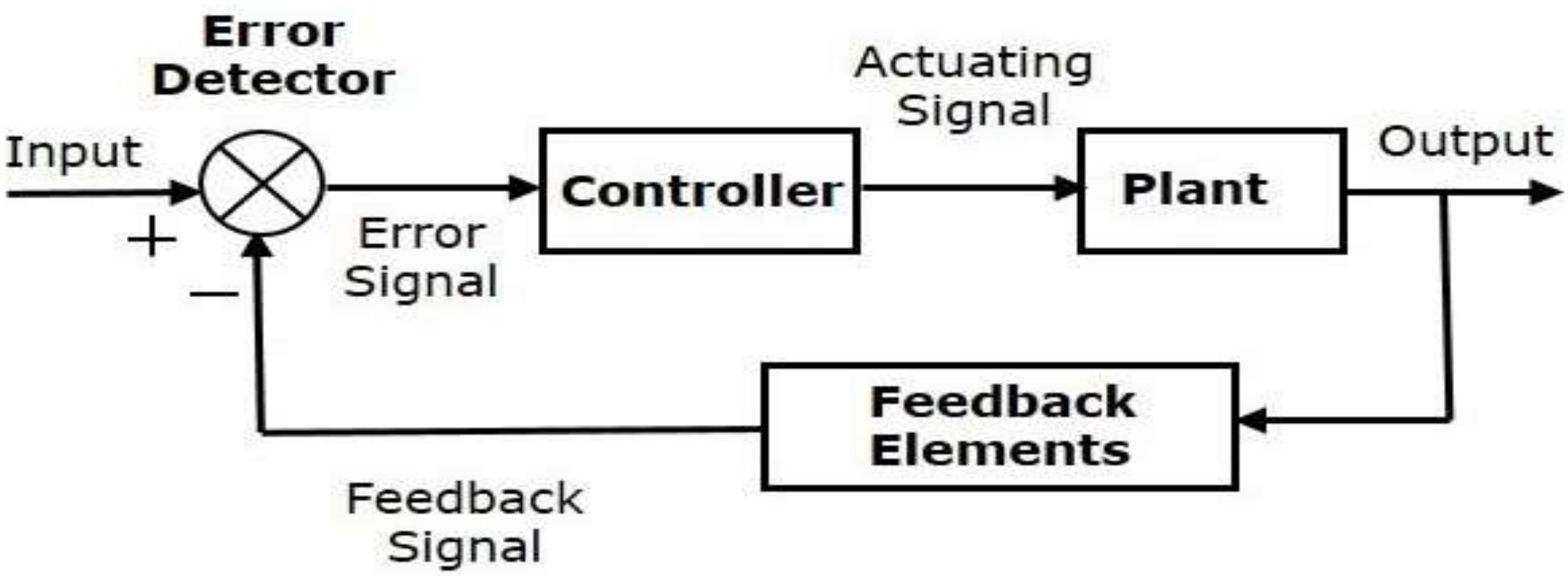




General Diagrams for Closed Loop Control Systems

Feedback from Output (Closed Loop System)

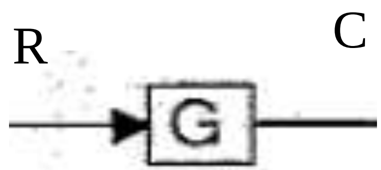




Effects of Negative Feedback Control Systems

Transfer Function / Gain of Negative Feed Back System

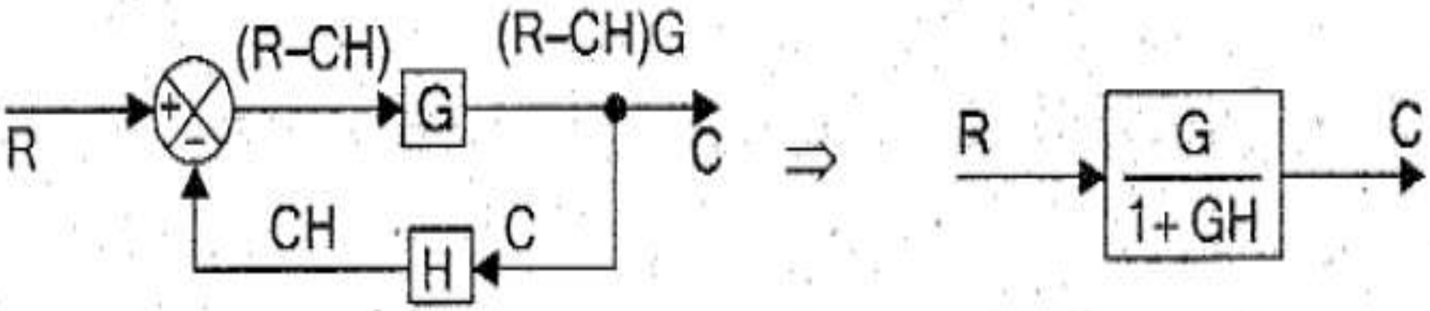
Open Loop System



$$\frac{C}{R} = G$$

Closed Loop System

Rule-10 : Elimination of (negative) feedback loop



Proof:

$$C = (R - CH)G \Rightarrow C = RG - CHG \Rightarrow C + CHG = RG$$

$$\therefore C(1 + HG) = RG \Rightarrow \frac{C}{R} = \frac{G}{1 + GH}$$

Effects of negative feedback on a system

1. Gain Decreases **[by factor (1+GH)]**

2. Bandwidth Increases **[by factor (1+GH)]**
   (Gain Bandwidth Produt is Constant)

3. Stability (Generally Stable system. In case of GH=-1 then unstable but it can be solved by cosidering a unity feedback)

4. Sensitivity (Less sensitive to parameter variations and disturbances)

Effect of Feedback on Gain

Open Loop System Gain is G

Closed Loop System Gain is G/(1+GH)

Overall Gain of system Decreases by a factor of (1+GH)

Effect of Feedback on Bandwidth

Gain Bandwidth Product is Constant

$$(\text{Gain})_{O.L}(\text{Bandwidth})_{O.L} = (\text{Gain})_{C.L}(\text{Bandwidth})_{C.L}$$

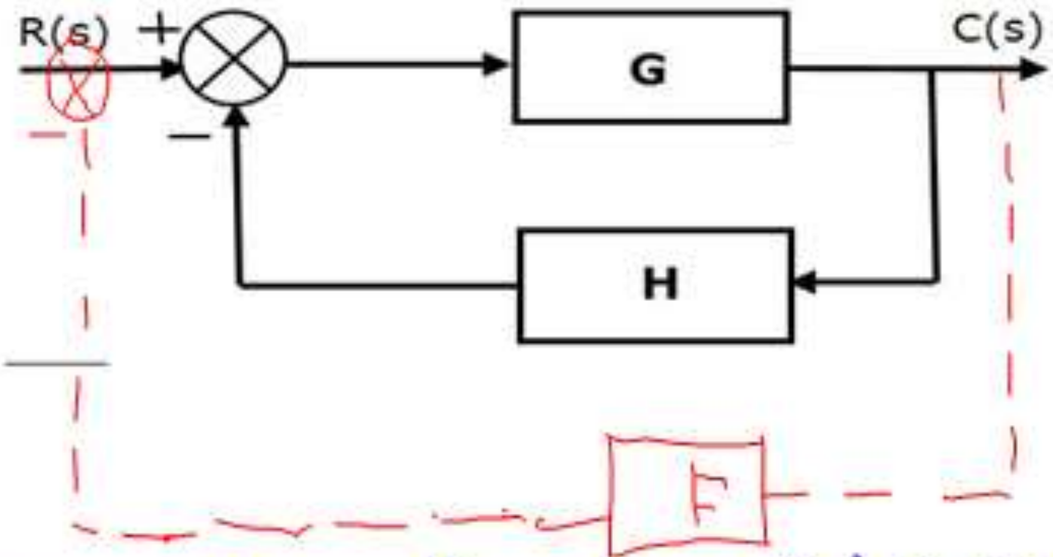
$$G.(B.W)_{OL} = \frac{G}{1+GH}(B.W)_{C.L}$$

$$(B.W)_{C.L} = (B.W)_{O.L}(1+GH)$$

Bandwidth of closed loop System = Bandwidth of Open loop System X (1+GH)

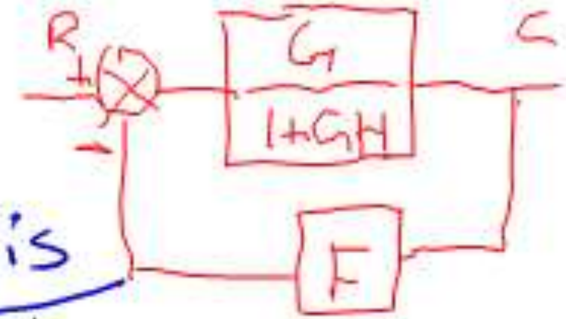
Bandwidth of system Increases by a factor of (1+GH)

Effect of Feedback on Stability



with out 'F' system Gain is

$$\left\{ \frac{G}{(1+GH)} \right.$$



with extra feedback 'F' Gain is

$$\left\{ \frac{C}{R} = \frac{\left(\dfrac{G}{1+GH}\right)}{1+\left(\dfrac{G}{1+GH}\right)F} = \frac{\left(\dfrac{G}{1+GH}\right)}{\left(\dfrac{1+GH+GF}{1+GH}\right)}\Bigg|_{GH=-1} = \frac{G}{GF} = \frac{1}{F} \right.$$

Effect of Feedback on Stability

If $GH = -1$ then the system

Gain $\frac{G}{1+GH}$ becomes $\frac{G}{1-1} = \frac{G}{0} = \infty$.

and system is unstable.

To make system stable a feedback with Gain F is added. Then the system Gain is $\frac{1}{F}$. Here system Gain is finite. Thus unstable system is changed to stable system using extra feedback

Effect of Feedback on Sensitivity due to G

$$Closed\ \ Loop\ \ T = \frac{G}{(1+GH)} \quad — ①$$

$$S_G^T = \frac{\frac{\partial T}{T}}{\frac{\partial G}{G}} = \frac{Percentage\ change\ in\ T}{Percentage\ change\ in\ G} \qquad S_G^T = \frac{\partial T}{\partial G}\frac{G}{T} \quad — ②$$

1) $$\frac{\partial T}{\partial G} = \frac{\partial}{\partial G}\left(\frac{G}{1+GH}\right) = \frac{(1+GH).1-G(H)}{(1+GH)^2} = \frac{1}{(1+GH)^2} \quad — ③$$

$$\frac{\partial}{\partial x}\left(\frac{u}{v}\right) = \frac{v\,du - u\,dv}{v^2}$$

2) From Eq ① $\frac{G}{T} = 1 + GH$ — ④

substitute ③ & ④ in ②

$$S_G^T = \frac{1}{(1+GH)^2}(1+GH) = \frac{1}{1+GH}$$

Effect of Feedback on Sensitivity

$$Open\,Loop \quad T = G \qquad — ①$$

$$S_G^T = \frac{\frac{\partial T}{T}}{\frac{\partial G}{G}} = \frac{Percentage\ change\ in\ T}{Percentage\ change\ in\ G} \qquad — ②$$

$$S_G^T = \frac{\partial T}{\partial G}\frac{G}{T} \qquad — ③$$

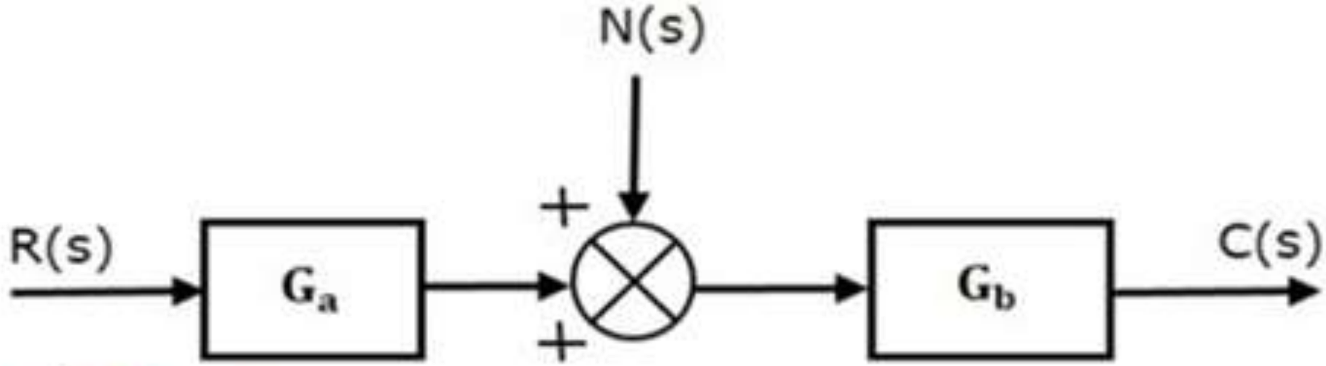
substitute ① in ③ $S_G^T = \dfrac{\partial G}{\partial G}\dfrac{G}{G} = 1$

Sensitivity of system Decreases and it is a function of G, H. That is closed loop system is less sensitive to parameter variations

Effect of Feedback Due to Noise / Disturbance

Open Loop System

N(s) – Noise

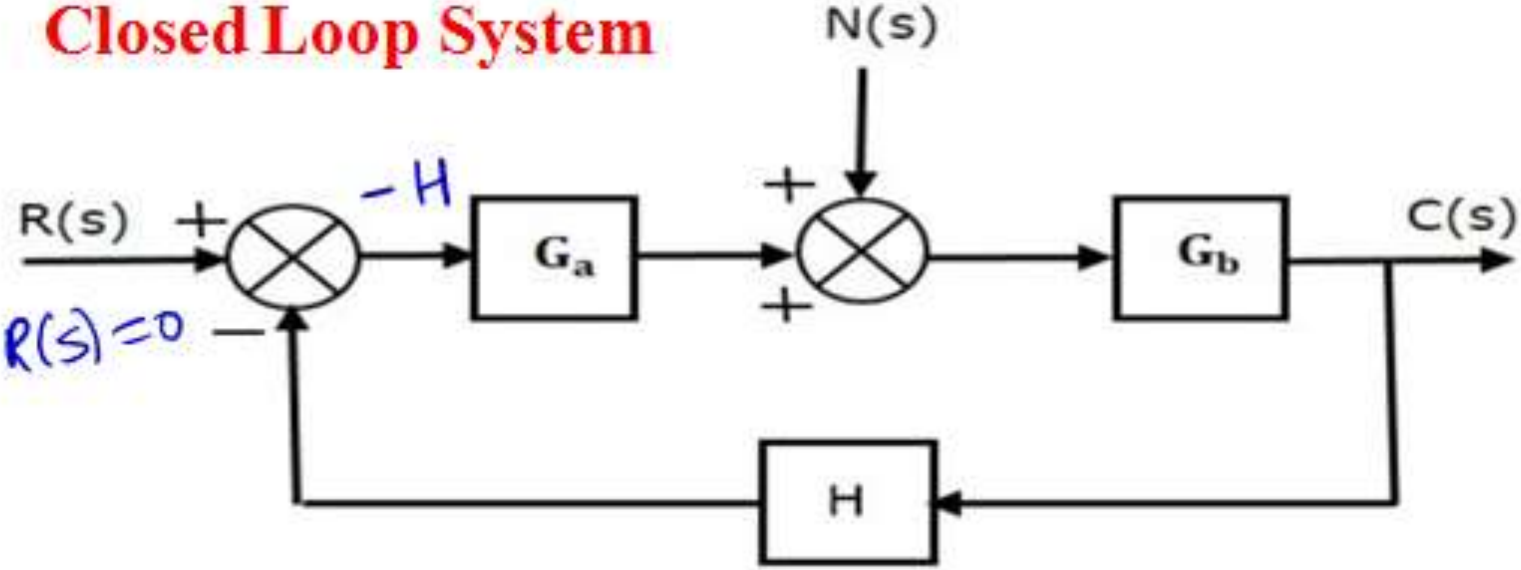


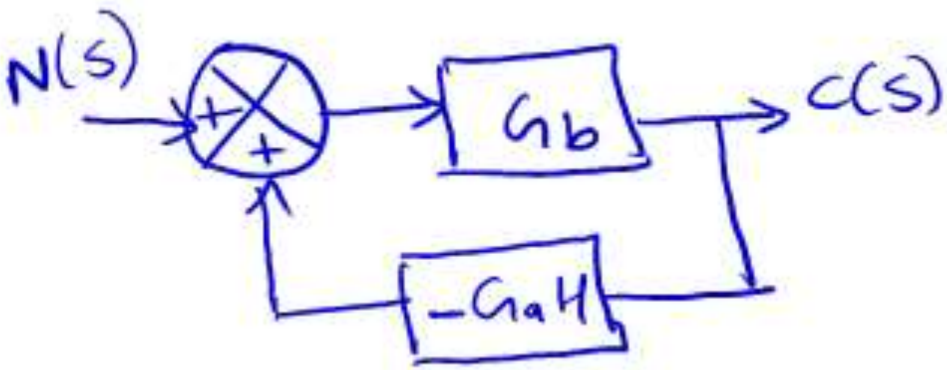
R(s) = 0
(Make other inputs to zero)

$$\frac{C(s)}{N(s)} = G_b$$

Effect of Feedback on Noise / Disturbance

Closed Loop System





$$\frac{C(s)}{N(s)} = \frac{G_b}{1+G_aG_bH}$$

Decrease by $(1+G_aG_bH)$

Effect of Noise can be decreased by using Feedback

Difference Between Open Loop and Closed Loop Control Systems

Advantages and Disadvantages of Open Loop and Closed Loop Control Systems

Open loop System Versus Closed Loop System

Open loop System	Closed Loop System
1. Controlling Process depends on only input and independent on output	1. Controlling Process depends on both input and output
2.Small Bandwidth	2.Large Bandwidth
3.Effect of parameter variations and noise is high	3.Effect of parameter variations and noise is Less
4..No Feedback	4. Feedback is present
5. Manual Controlling	5. Automatic Controlling
6. Error Detetor is Not Present	6. Error Detector is Present
7. It is Stable	7. Positive feedback not stable (Oscillatory) Negative feedback Stable (If require additional feedback is used)
8. Cheap and Economical	8. Costlier
9.Easy to Construct	9. Difficult to Construct
10. Less Maintenance	10. More Maintenance
11. Not Accurate and Not Reliable	11. Accurate and Reliable

Advantages of open loop systems

1. The open loop systems are simple and economical.
2. The open loop systems are easier to construct.
3. Generally the open loop systems are stable.

Disadvantages of open loop systems

1. The open loop systems are inaccurate and unreliable.
2. The changes in the output due to external disturbances are not corrected automatically.

Advantages of closed loop systems

1. The closed loop systems are accurate.
2. The closed loop systems are accurate even in the presence of non-linearities.
3. The sensitivity of the systems may be made small to make the system more stable.
4. The closed loop systems are less affected by noise.

Disadvantages of closed loop systems

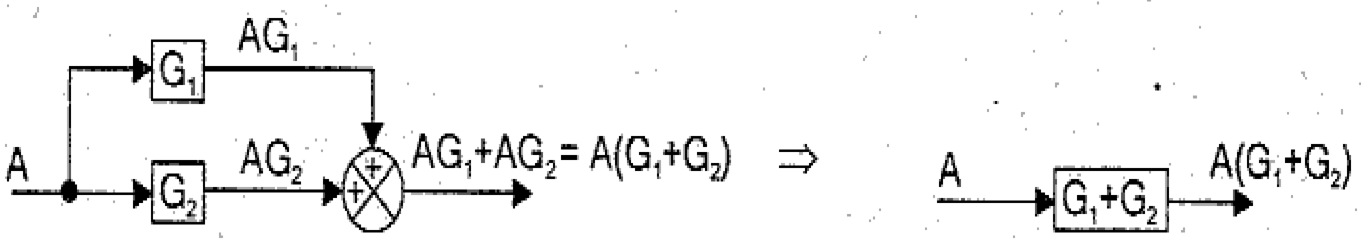
1. The closed loop systems are complex and costly.
2. The feedback in closed loop system may lead to oscillatory response.
3. The feedback reduces the overall gain of the system.
4. Stability is a major problem in closed loop system and more care is needed to design a stable closed loop system.

Block Diagram Reduction Algebra for obtaining the Transfer Function

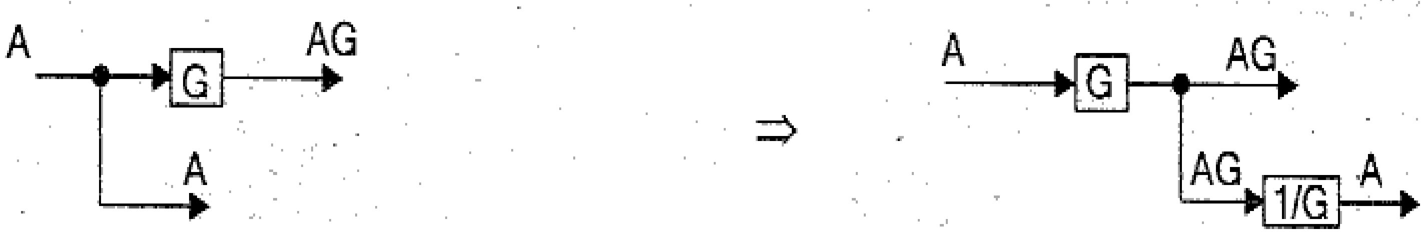
Rules for Block Diagram Reduction

Rule-1 : ***Combining the blocks in cascade***

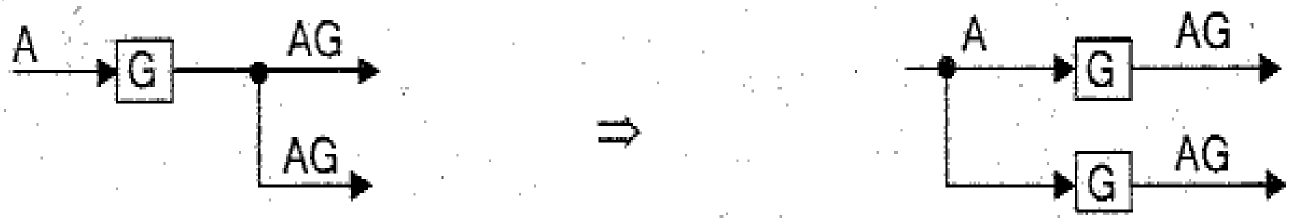
Rule-2 : ***Combining Parallel blocks (or combining feed forward paths)***



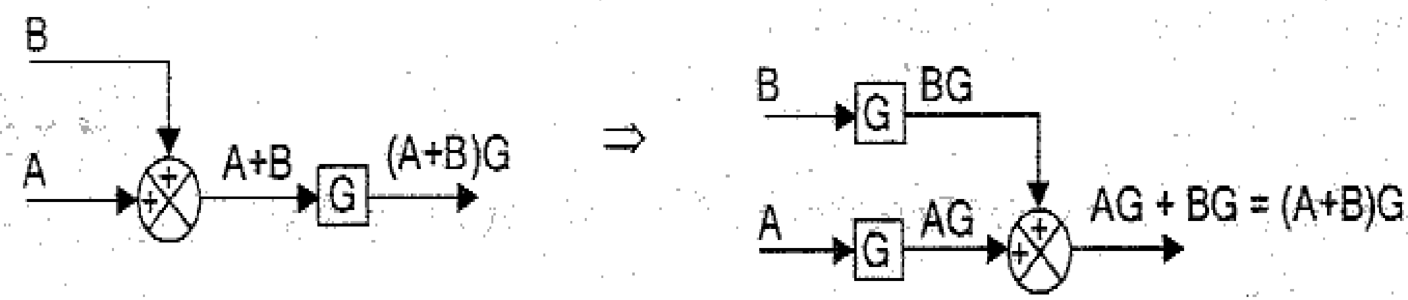
Rule-3 : ***Moving the branch point ahead of the block***



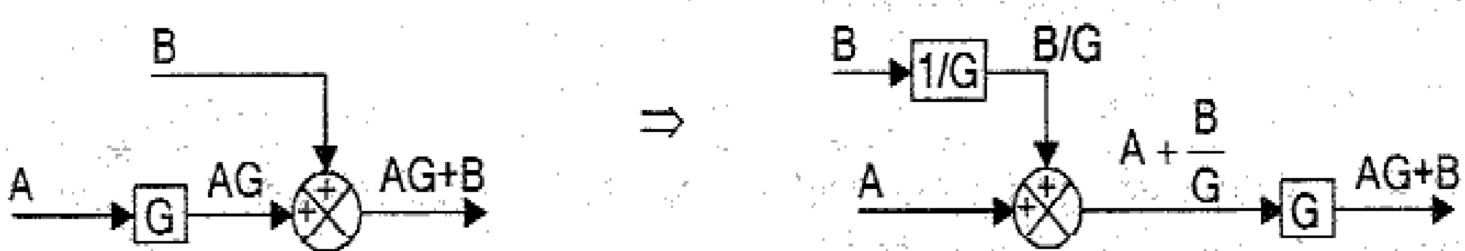
Rule-4 : ***Moving the branch point before the block***



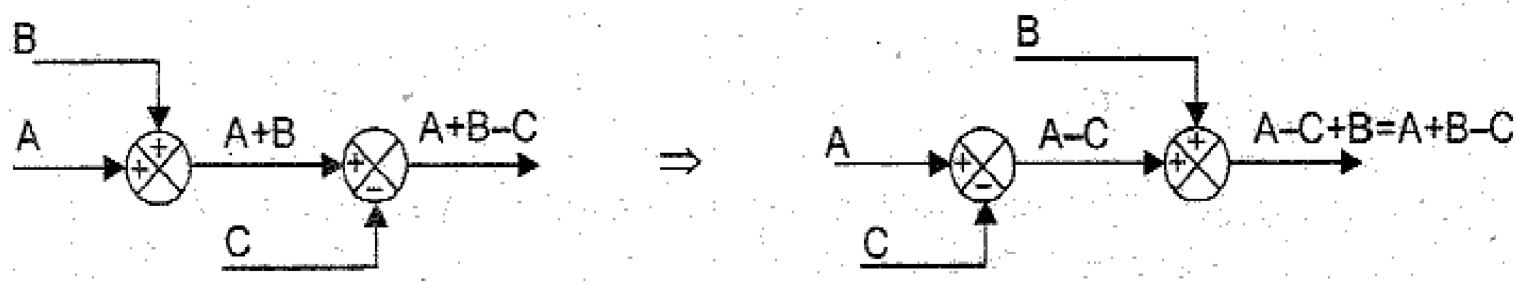
Rule-5 : ***Moving the summing point ahead of the block***



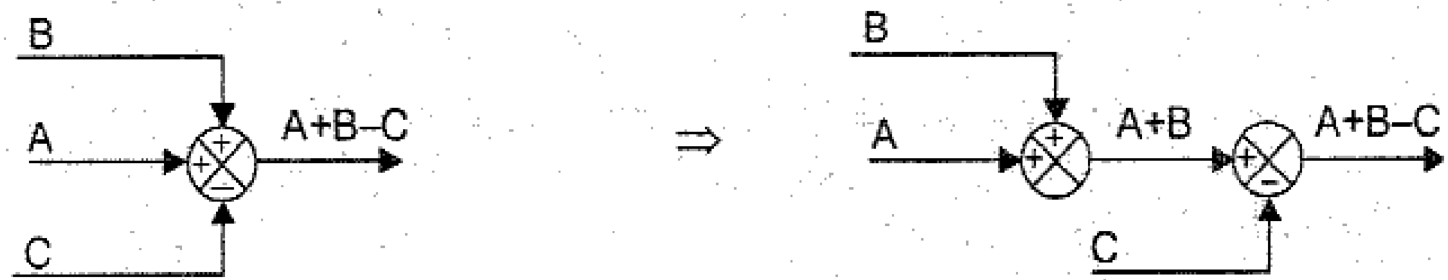
Rule-6 : ***Moving the summing point before the block***



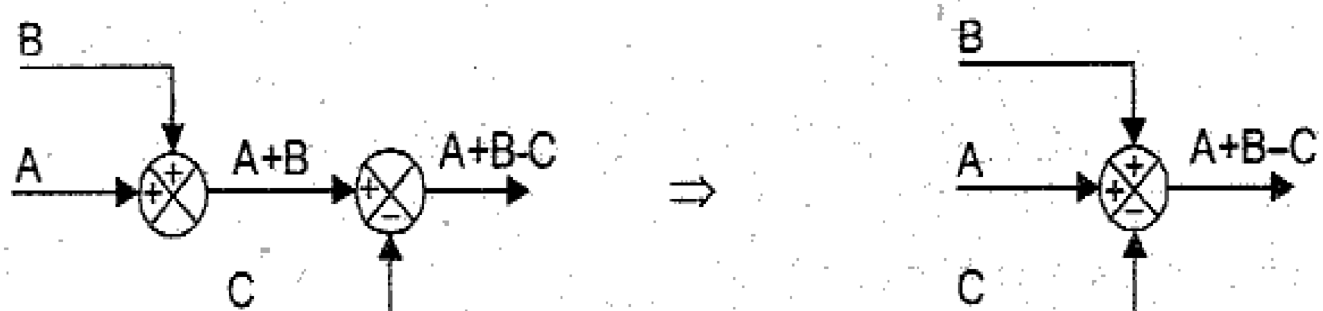
Rule-7 : Interchanging summing point



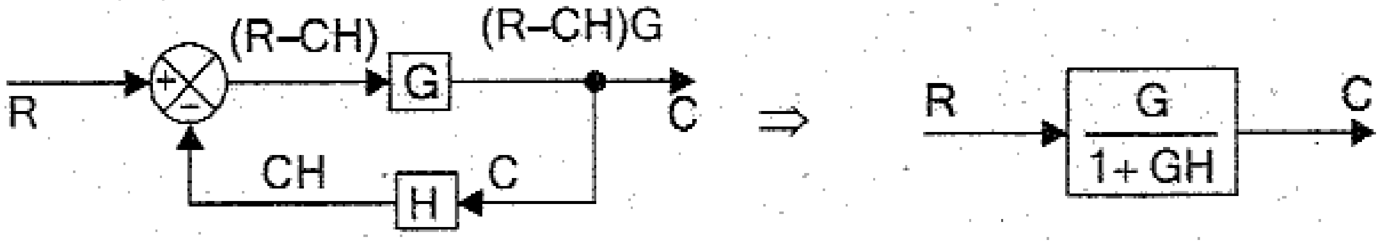
Rule-8 : Splitting summing points



Rule-9 : Combining summing points



Rule-10 : Elimination of (negative) feedback loop

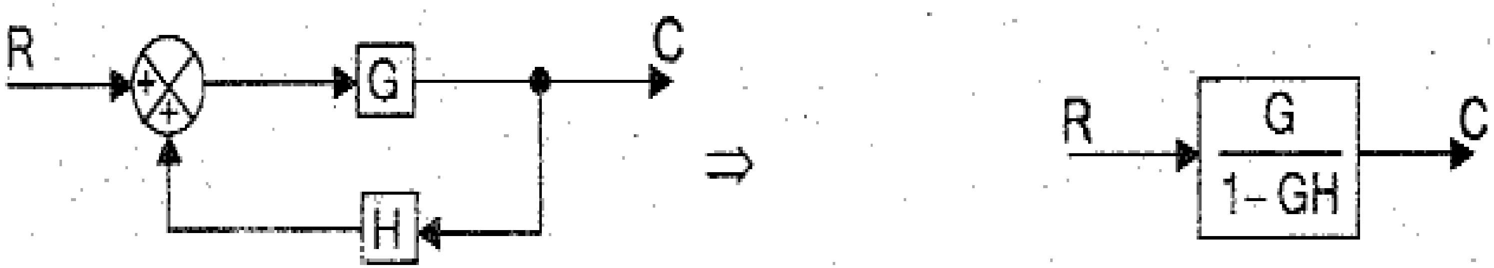


Proof:

$$C = (R - CH)G \quad \Rightarrow \quad C = RG - CHG \quad \Rightarrow \quad C + CHG = RG$$

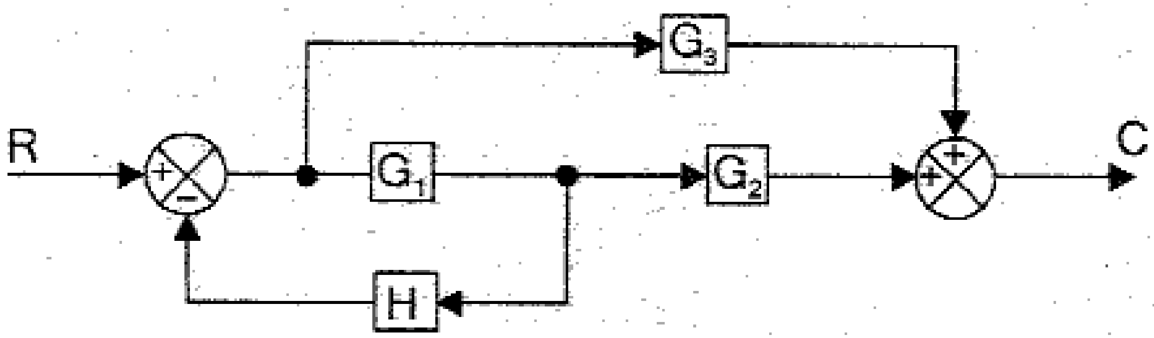
$$\therefore C(1 + HG) = RG \quad \Rightarrow \quad \frac{C}{R} = \frac{G}{1 + GH}$$

Rule-11 : Elimination of (positive) feedback loop

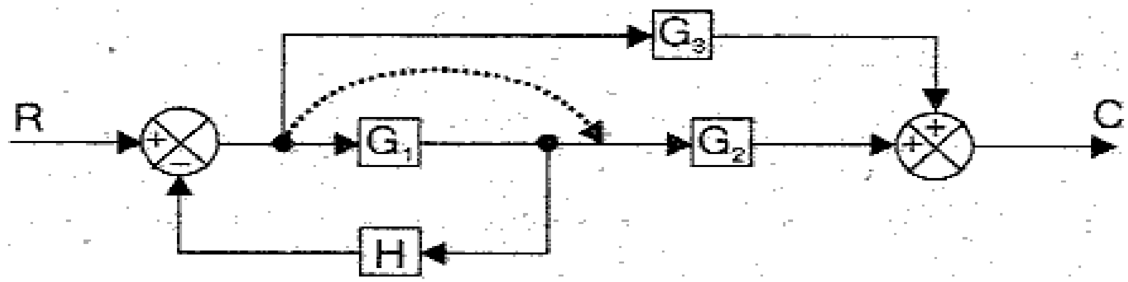


Problem 1 (Method 1)

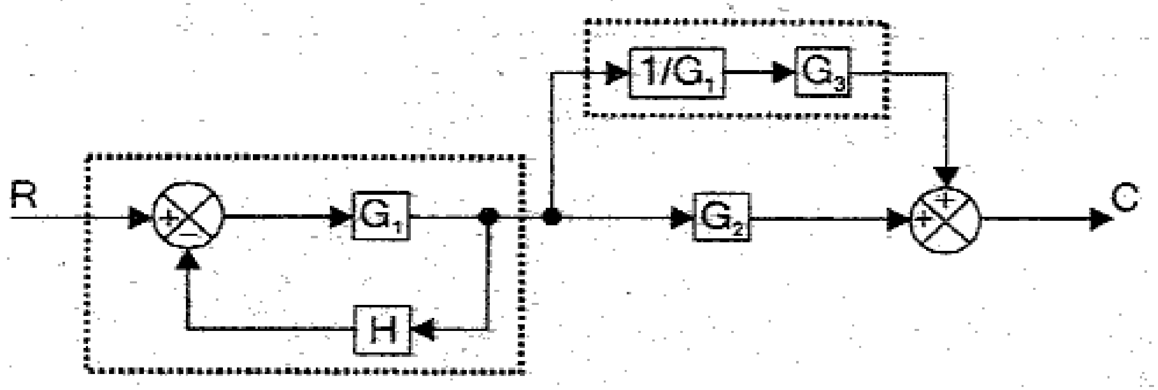
Reduce the block diagram shown in fig 1 and find C/R.



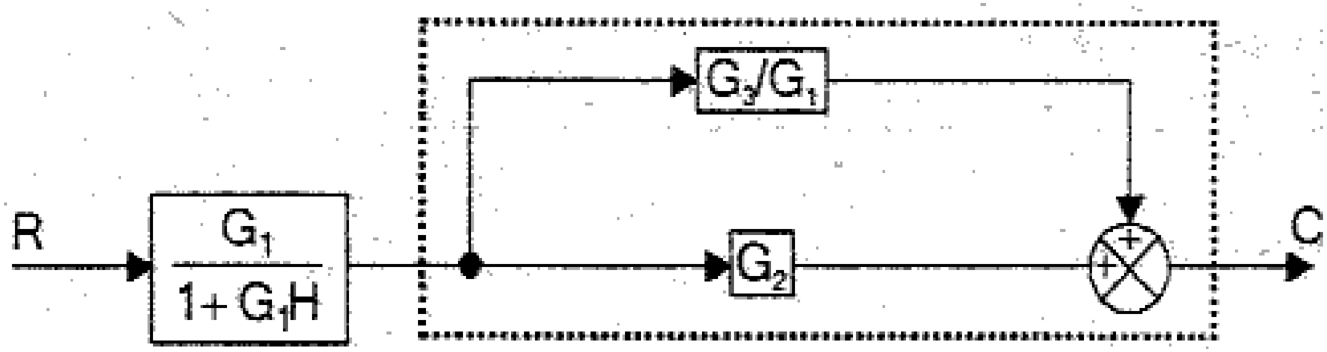
Step 1: Move the branch point after the block.



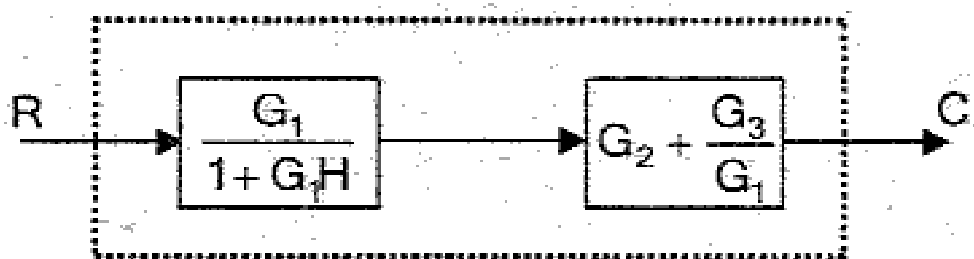
Step 2: Eliminate the feedback path and combining blocks in cascade.



Step 3: Combining parallel blocks



Step 4: Combining blocks in cascade

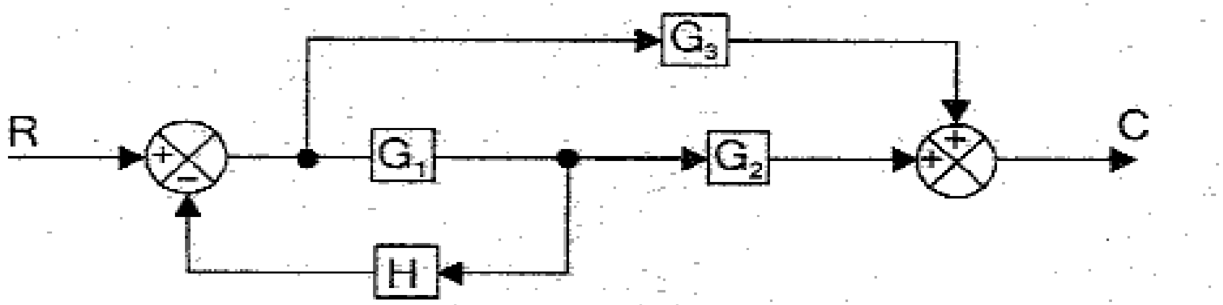


$$\frac{C}{R}=\left(\frac{G_1}{1+G_1H}\right)\left(G_2+\frac{G_3}{G_1}\right)=\left(\frac{G_1}{1+G_1H}\right)\left(\frac{G_1G_2+G_3}{G_1}\right)=\frac{G_1G_2+G_3}{1+G_1H}$$

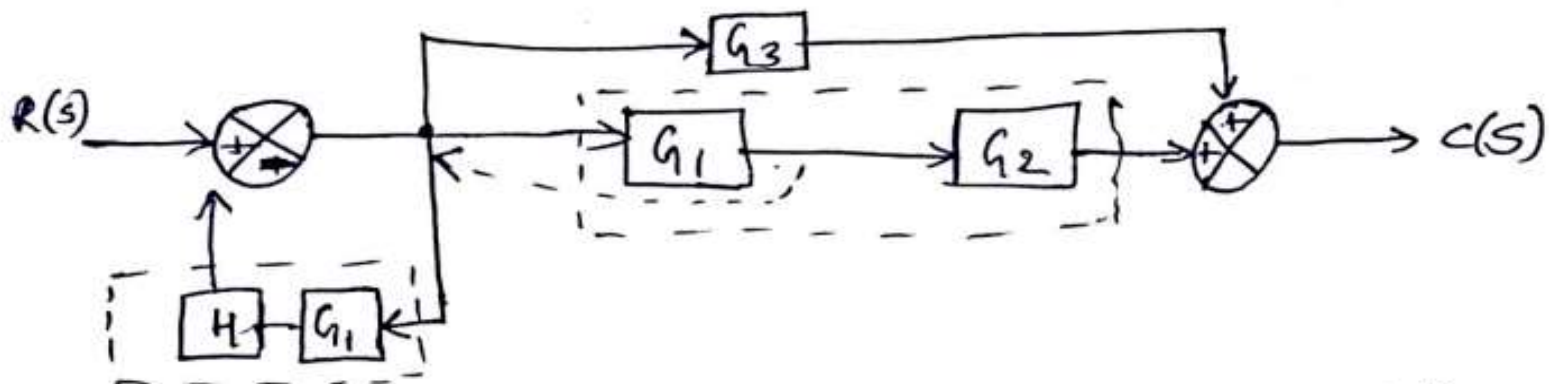
The overall transfer function of the system, $\frac{C}{R}=\frac{G_1G_2+G_3}{1+G_1H}$

Problem 1 (Method 2)

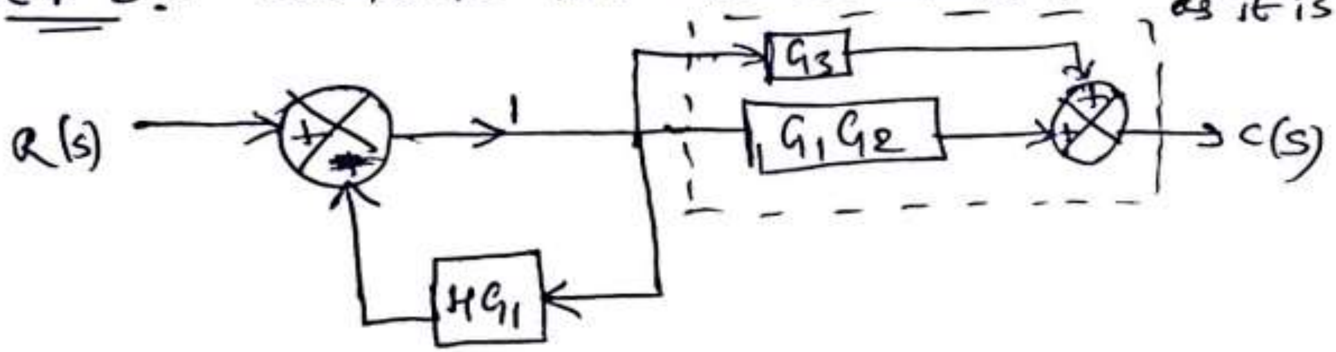
Reduce the block diagram shown in fig 1 and find C/R.



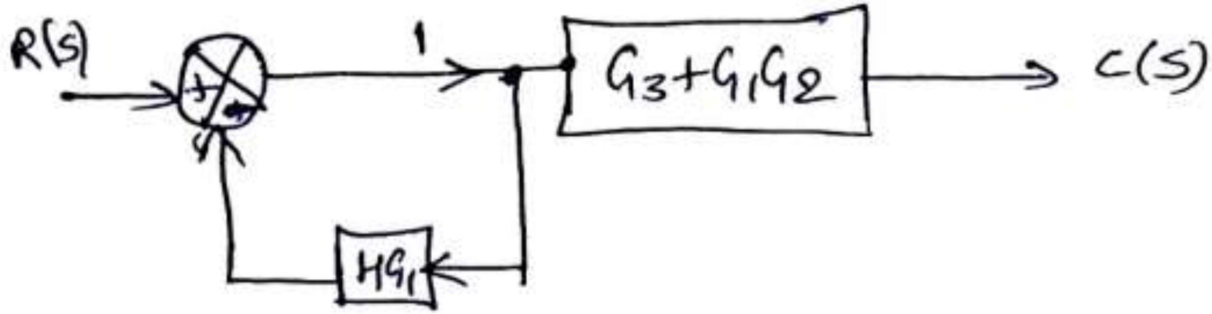
Step 1 :- Move the branch point before the block



step 2 :- combine Two cascade blocks { Assume Gain $\xrightarrow{1}$ as it is straight path }

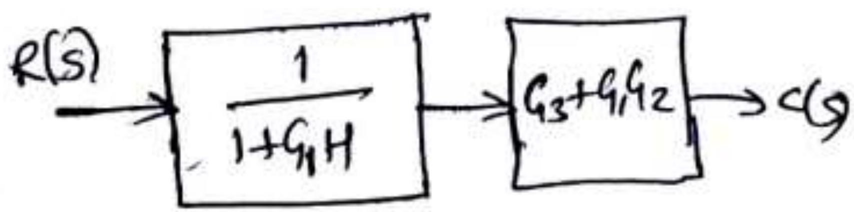


Step 3:- combine two parallel blocks

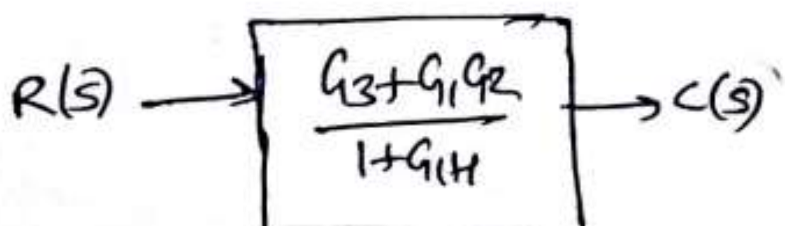


Step 4:- Eliminate −ve feed back loop. Here $G(s)=1$, $H(s)=G_1H$. $\frac{G(s)}{1+G(s)H(s)}$

Then $\Rightarrow \frac{1}{1+(1)G_1H}$



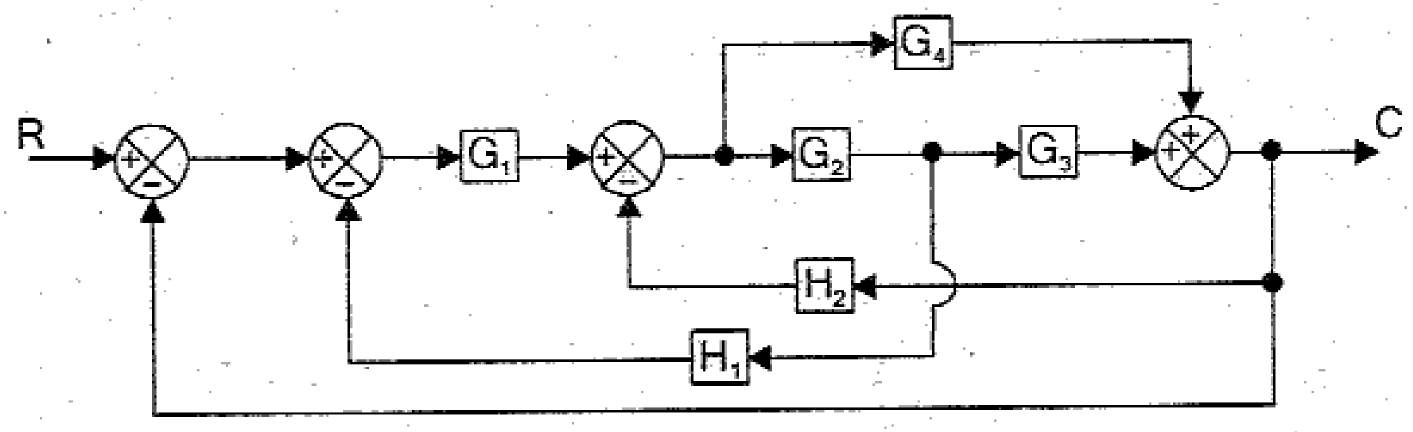
Step 5:- combine cascade blocks.



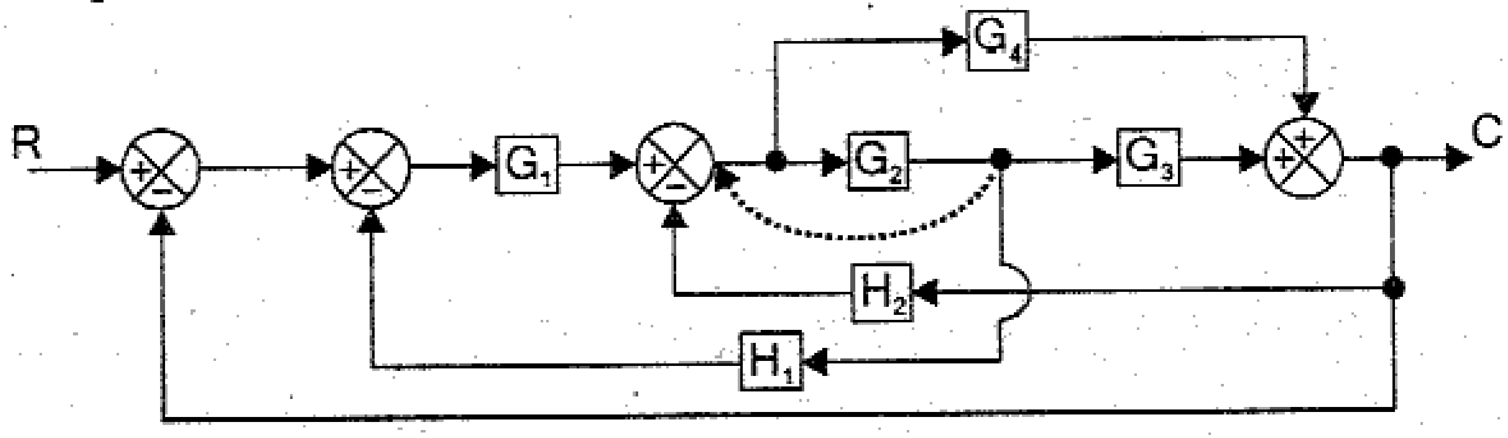
$$\frac{C(S)}{R(S)} = \frac{G_3+G_1G_2}{1+G_1H}$$

Problem 2 (Method 1)

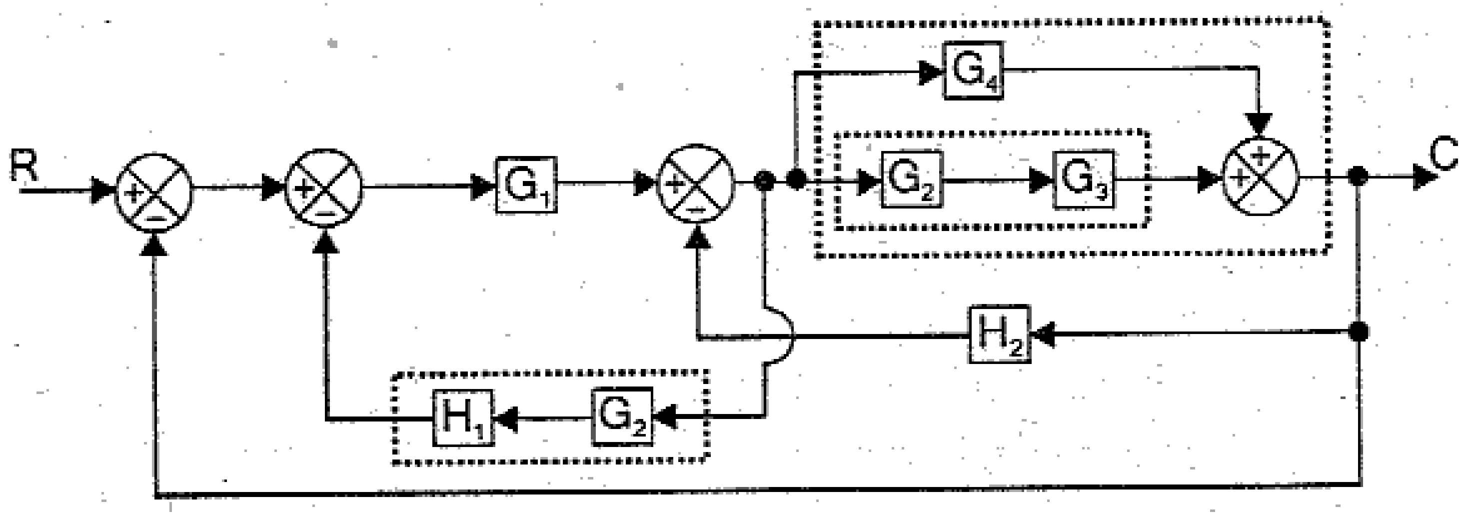
Reduce the block diagram shown in fig 1 and find C/R.



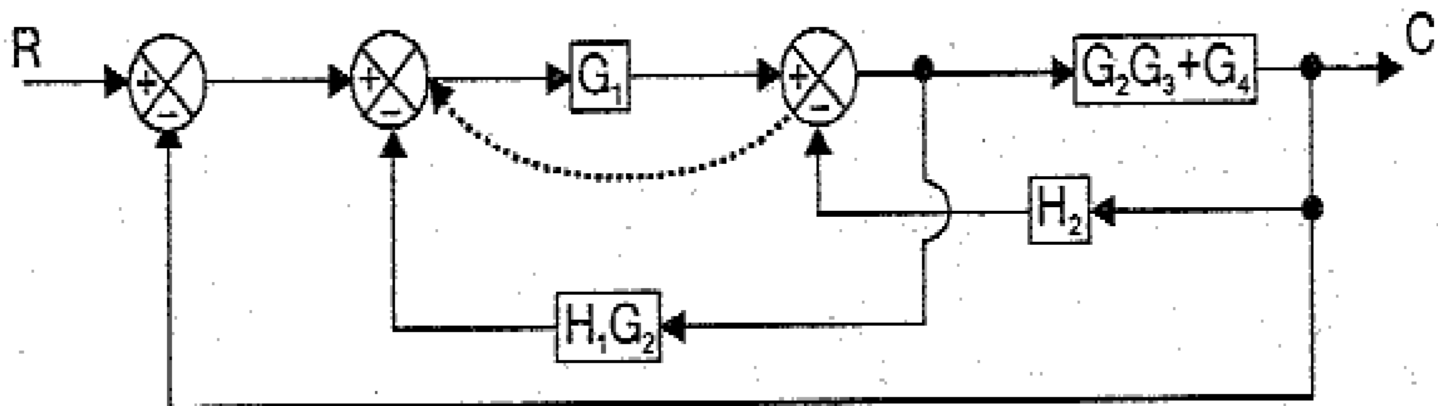
Step 1: Moving the branch point before the block



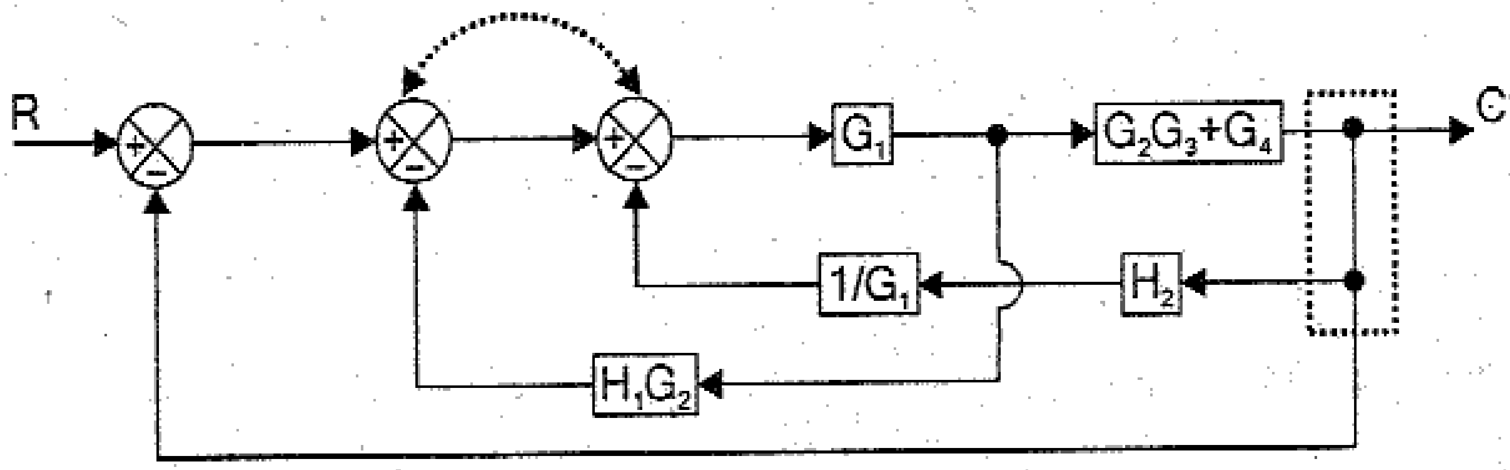
Step 2: Combining the blocks in cascade and eliminating parallel blocks



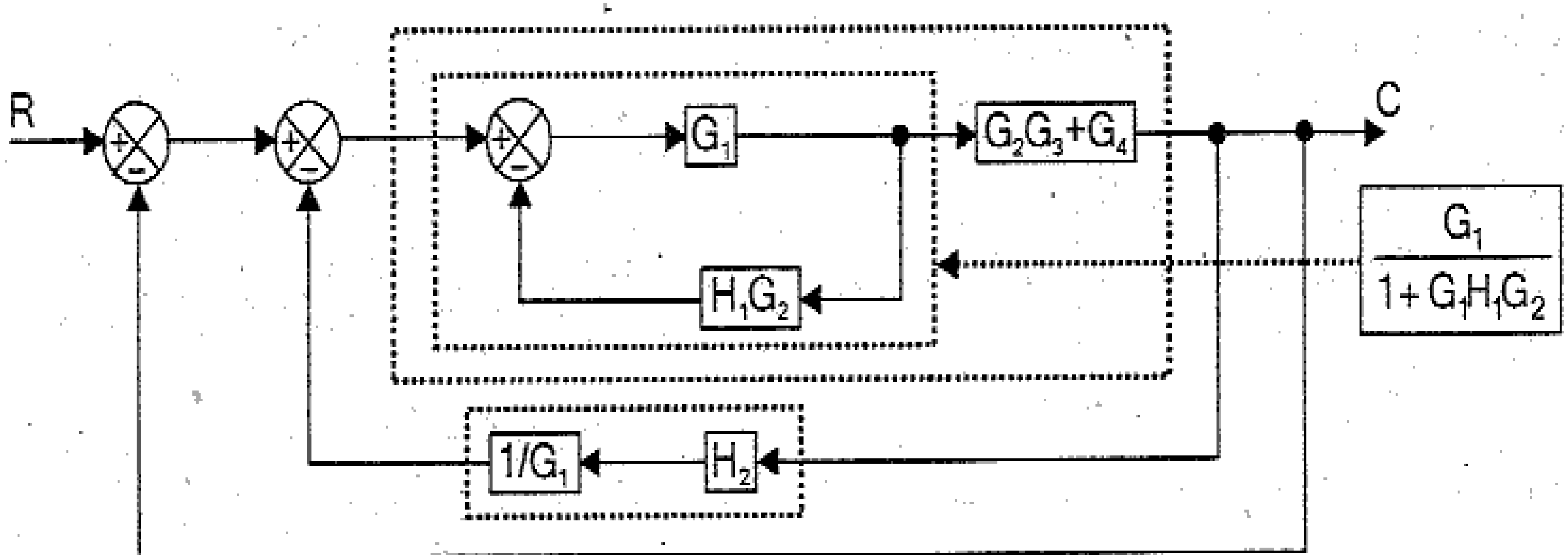
Step 3: Moving summing point before the block.



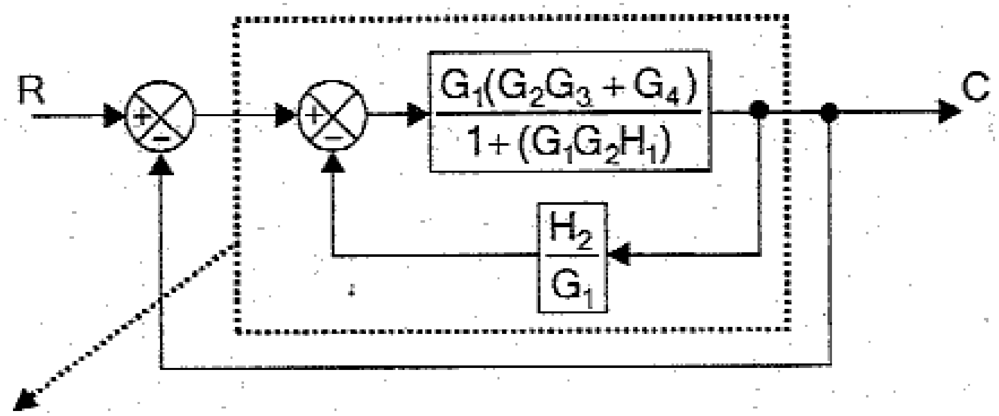
Step 4: Interchanging summing points and modifying branch points.



Step 5: Eliminating the feedback path and combining blocks in cascade

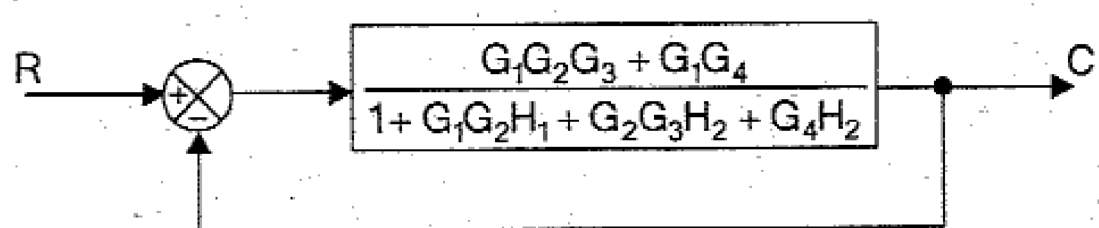


Step 6: Eliminating the feedback path



$$\frac{\frac{G_1(G_2G_3+G_4)}{1+G_1G_2H_1}}{1+\frac{G_1(G_2G_3+G_4)}{1+G_1G_2H_1}\frac{H_2}{G_1}} \Rightarrow \frac{\frac{G_1G_2G_3+G_1G_4}{1+G_1G_2H_1}}{\frac{1+G_1G_2H_1+G_2G_3H_2+G_4H_2}{1+G_1G_2H_1}} \Rightarrow \frac{G_1G_2G_3+G_1G_4}{1+G_1G_2H_1+G_2G_3H_2+G_4H_2}$$

Step 7: Eliminating the feedback path

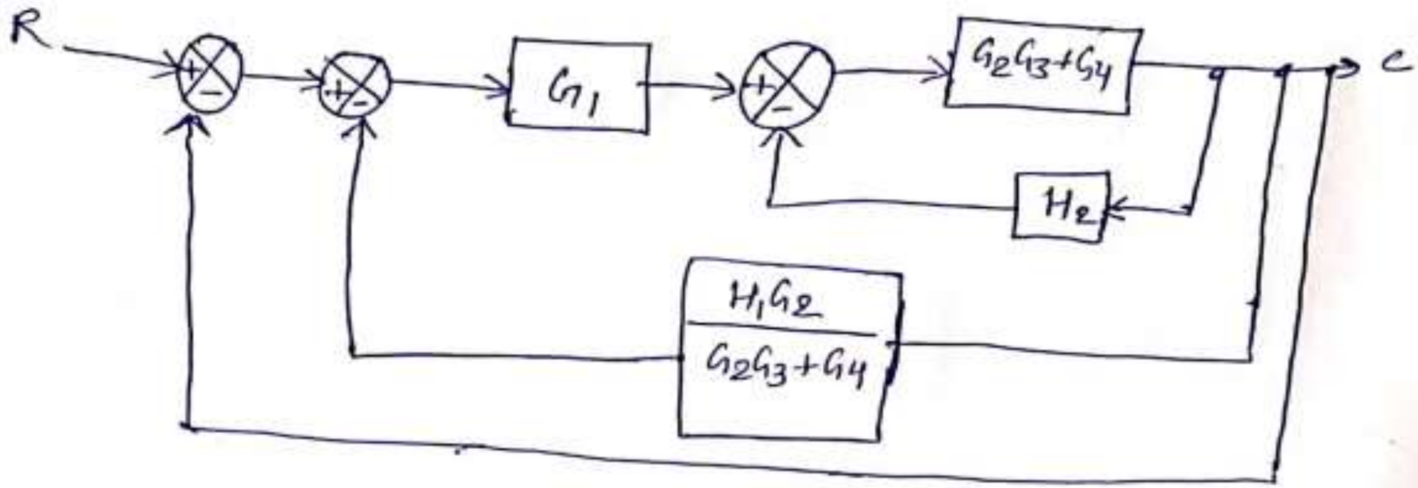


$$\frac{C}{R} = \frac{\frac{G_1G_2G_3+G_1G_4}{1+G_1G_2H_1+G_2G_3H_2+G_4H_2}}{1+\frac{G_1G_2G_3+G_1G_4}{1+G_1G_2H_1+G_2G_3H_2+G_4H_2}} = \frac{G_1G_2G_3+G_1G_4}{1+G_1G_2H_1+G_2G_3H_2+G_4H_2+G_1G_2G_3+G_1G_4}$$

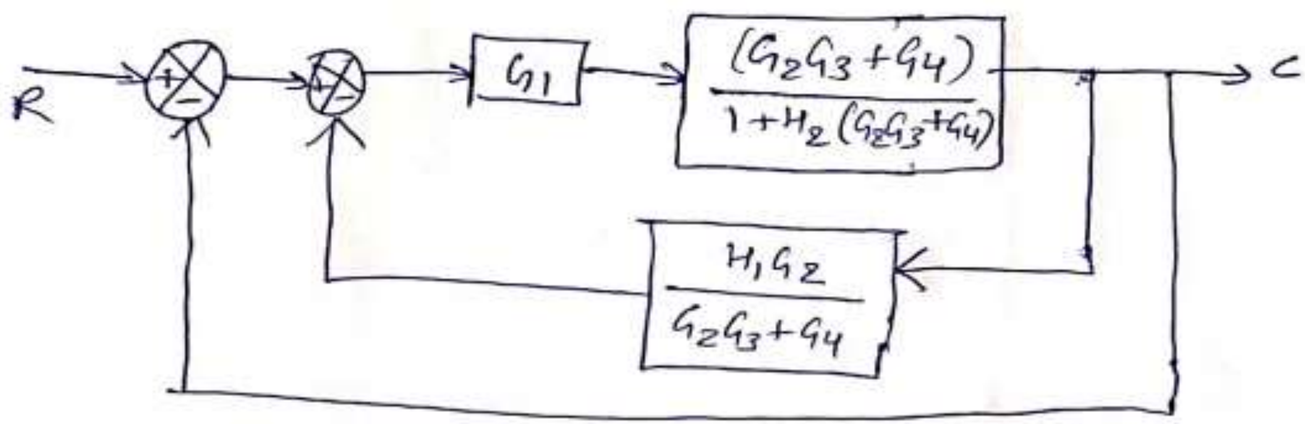
Problem 2 (Method 2)

1st & Step2:- same

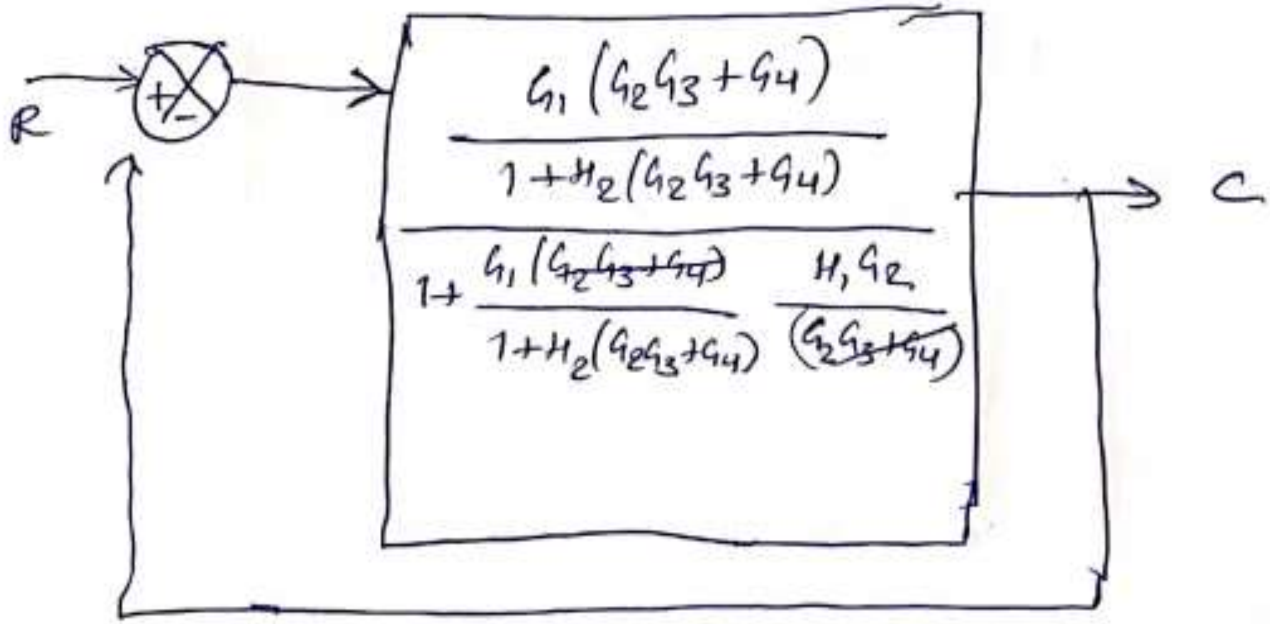
step3:- Moving Branch point after block $(G_2G_3+G_4)$ and split Branch points at output node



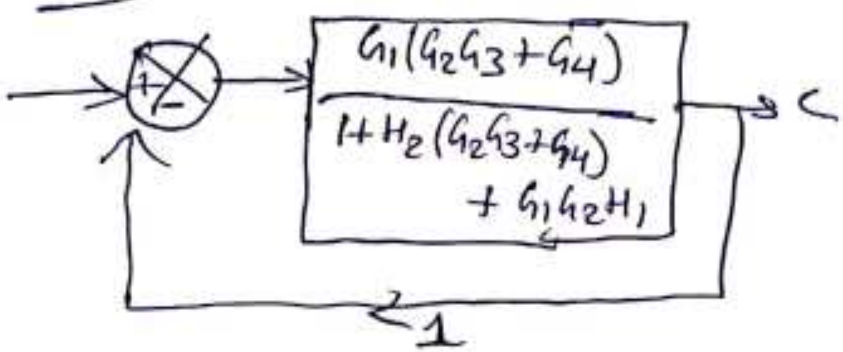
step4:- Eliminate feedback loops



Step 5:- Combine Cascade Block and Eliminate feedback



Step 6: Eliminate feedback after simplification

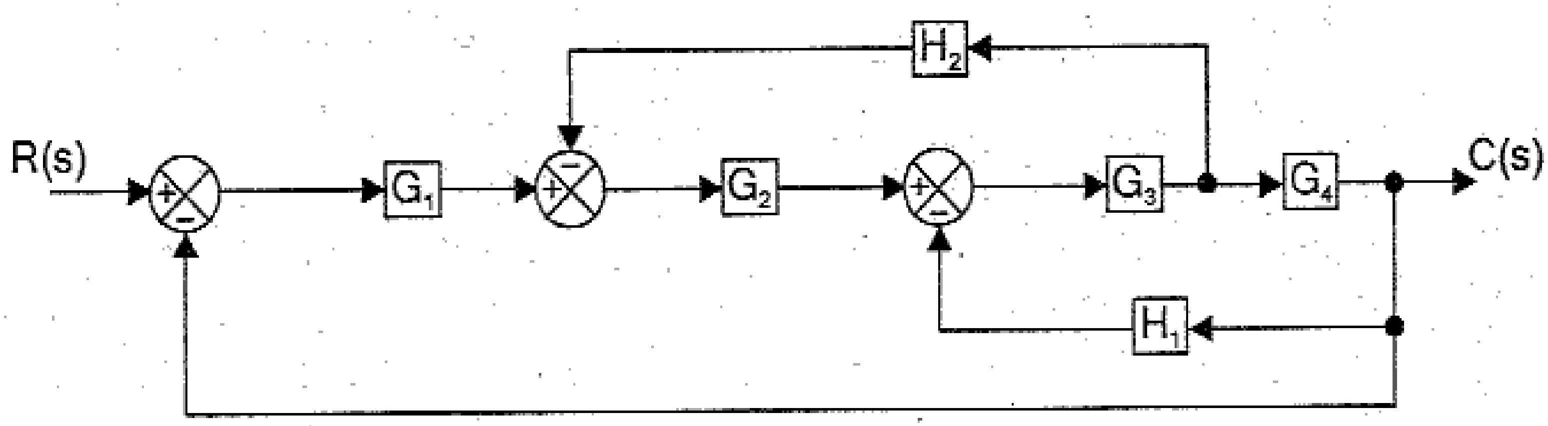


$$\Rightarrow \frac{G_1G_2G_3+G_1G_4}{1+G_1G_2H_1+G_2G_3H_2+G_4H_2+G_1G_2G_3+G_1G_4}$$

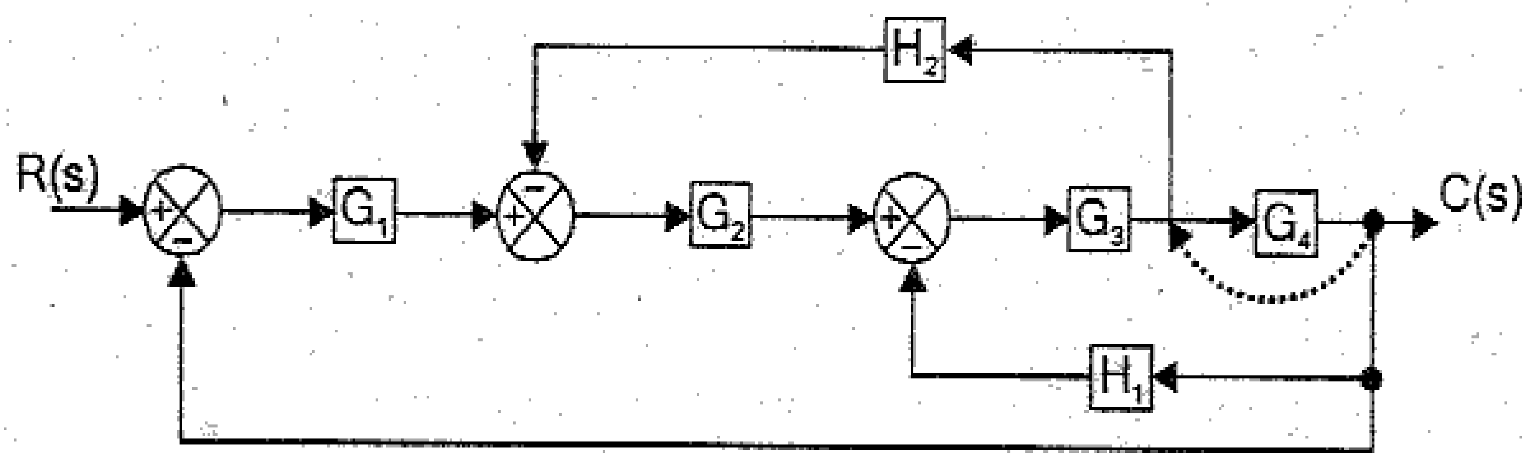
$=$

Problem 3 (Method 1)

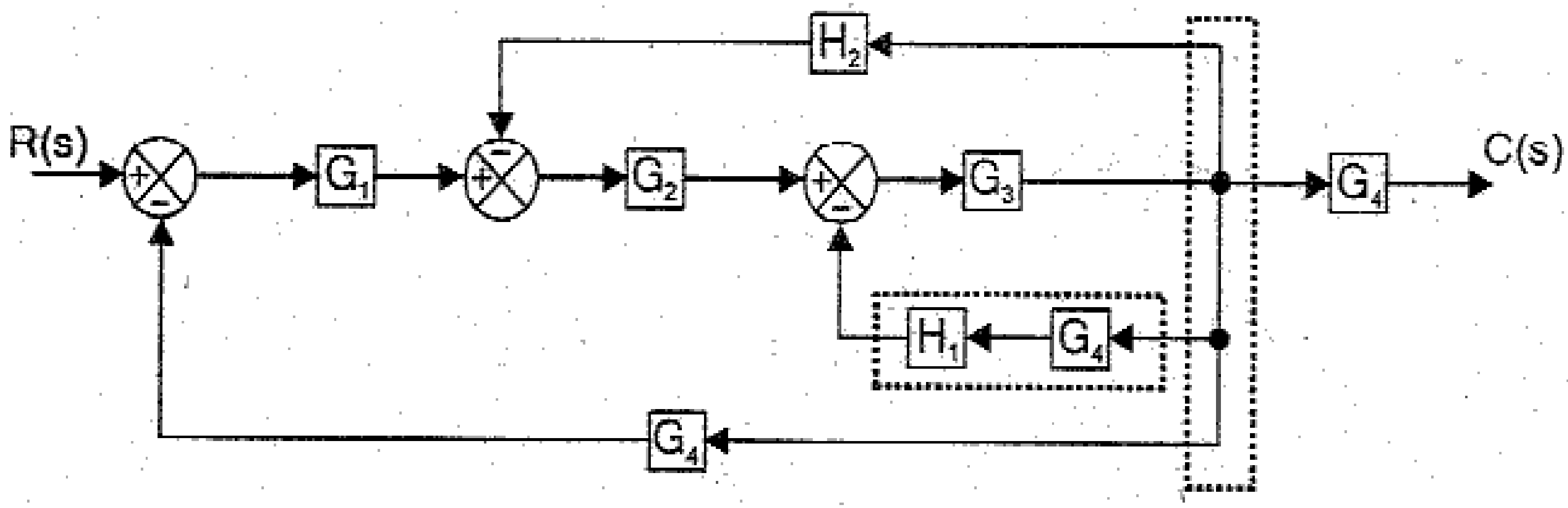
Determine the overall transfer function $\frac{C(s)}{R(s)}$ for the system shown in fig 1.



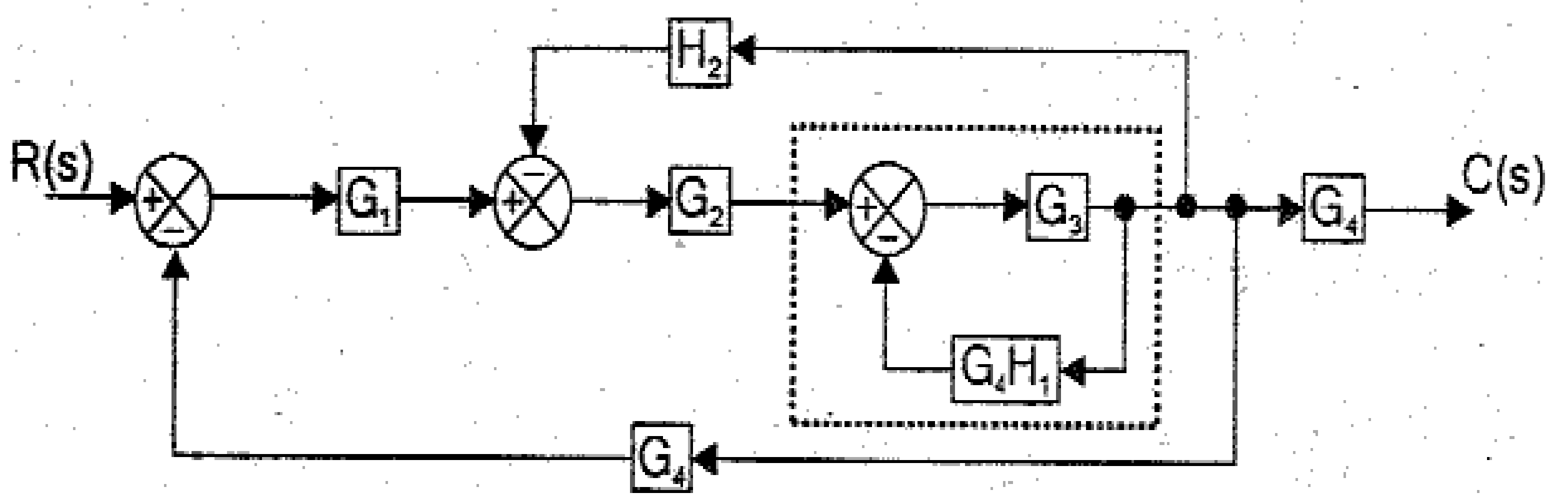
Step 1: Moving the branch point before the block



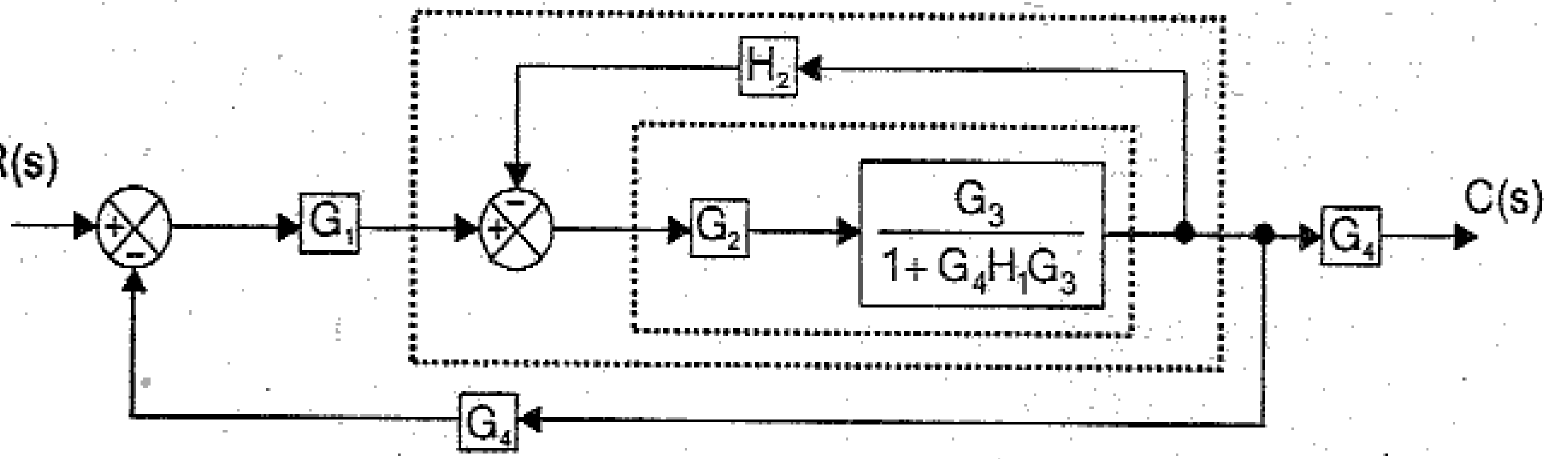
Step 2 : Combining the blocks in cascade and rearranging the branch points



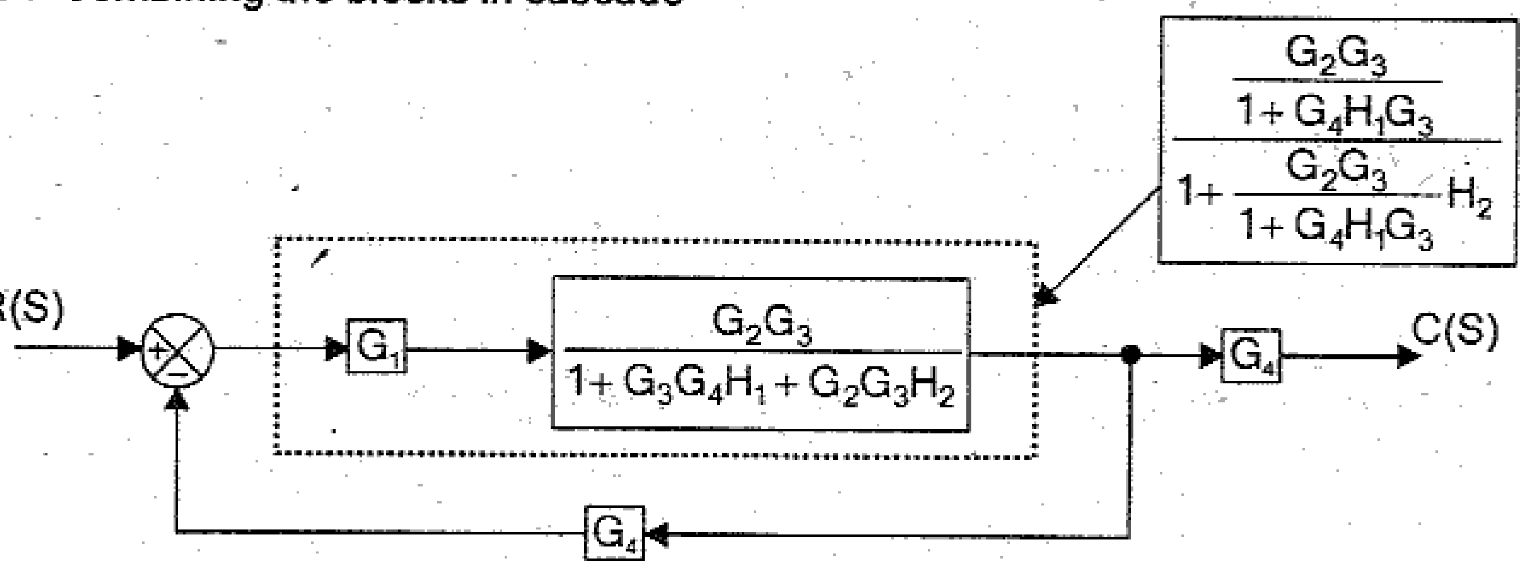
Step 3 : Eliminating the feedback path



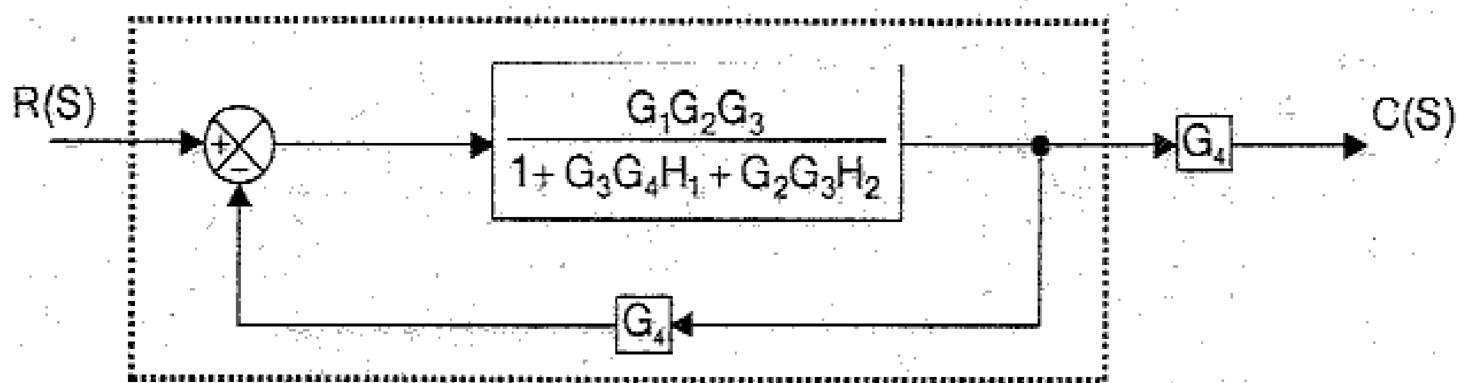
Step 4 : Combining the blocks in cascade and eliminating feedback path



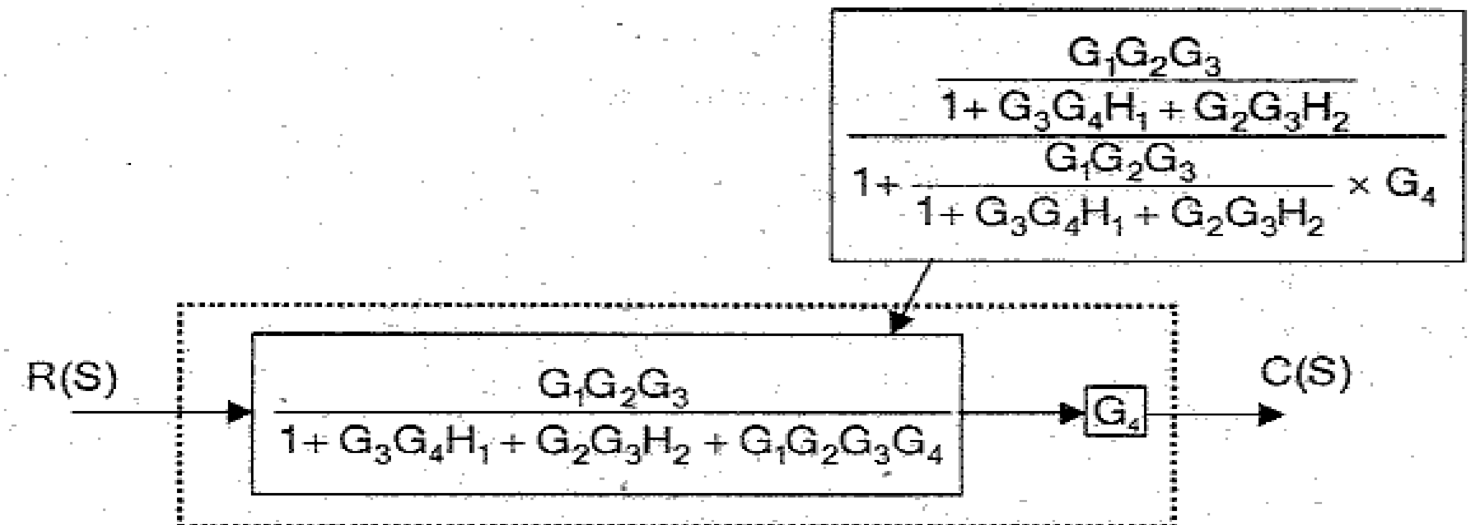
Step 5 : Combining the blocks in cascade



Step 6 : Eliminating the feedback path

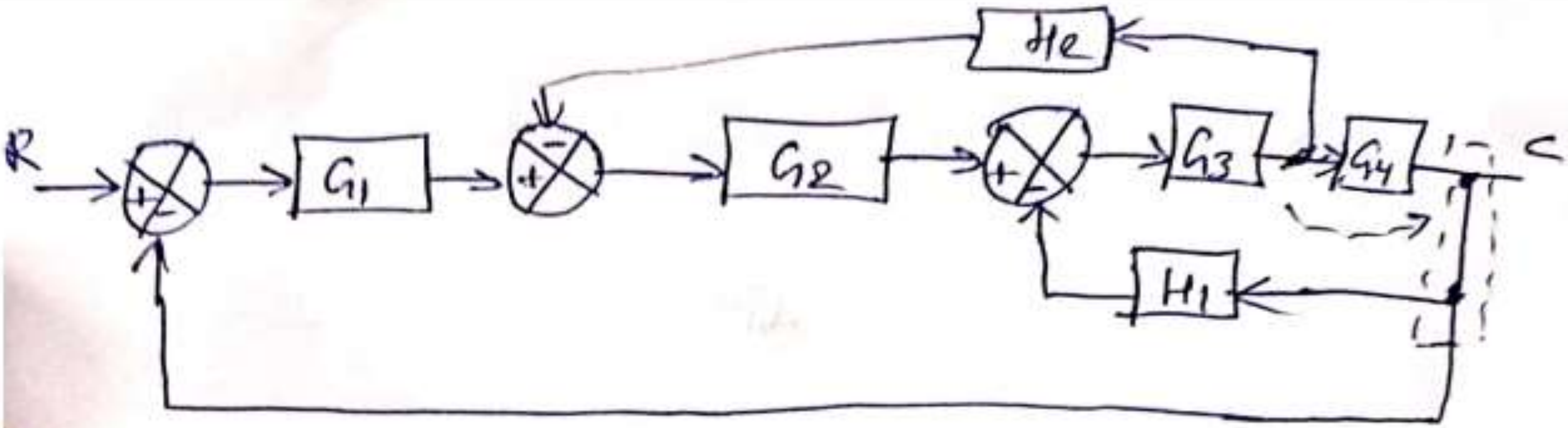


Step 7 : Combining the blocks in cascade

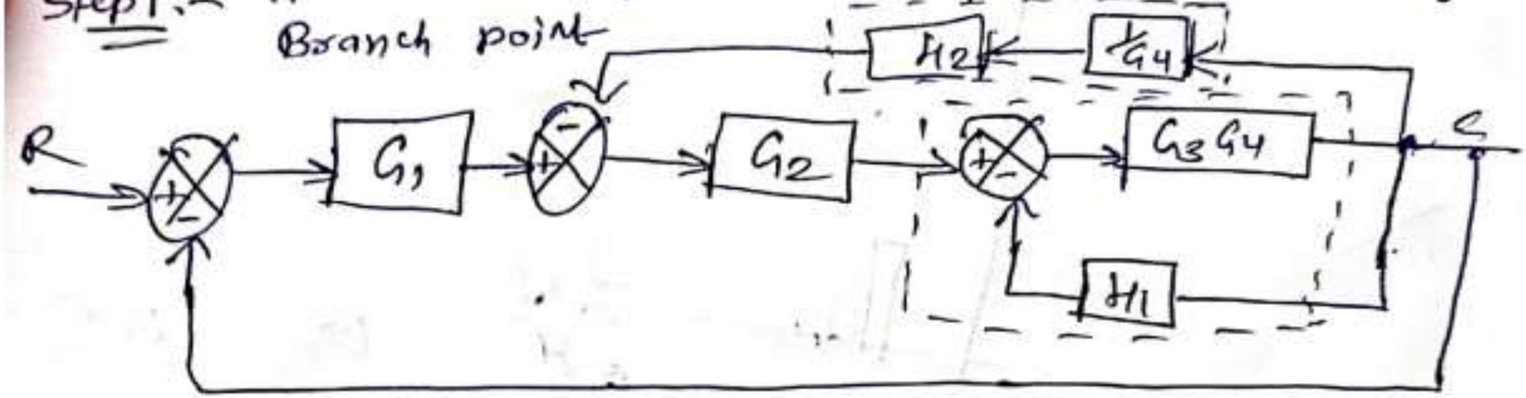


$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4}{1+G_3G_4H_1+G_2G_3H_2+G_1G_2G_3G_4}$$

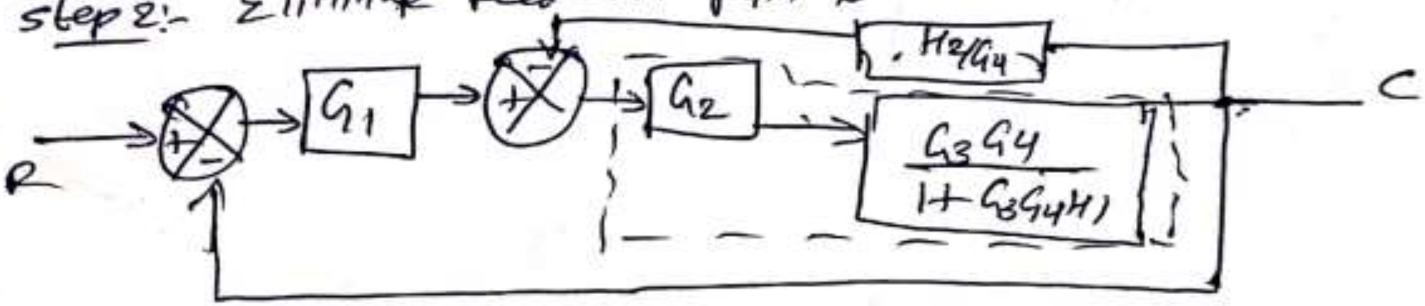
Problem 3 (Method 2)



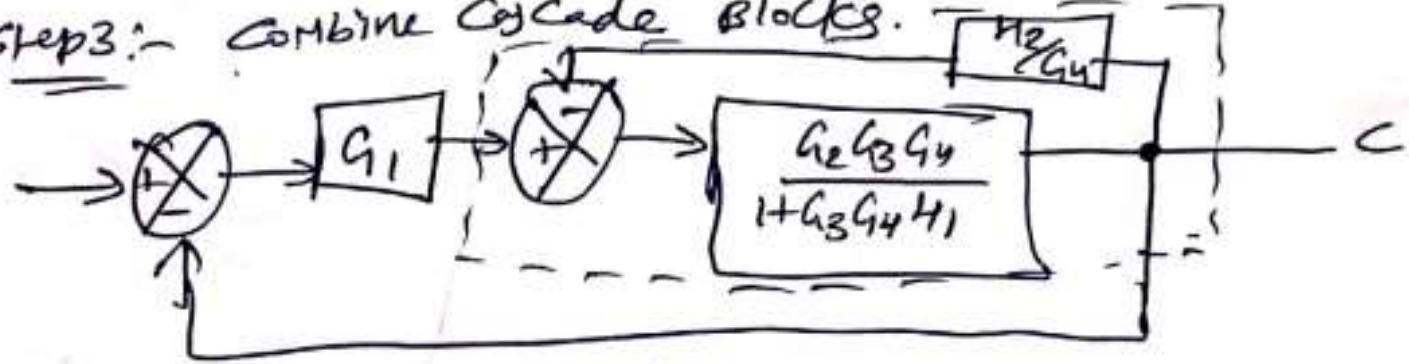
Step 1:- Move Branch point after G_4 and Rearrange of Branch point



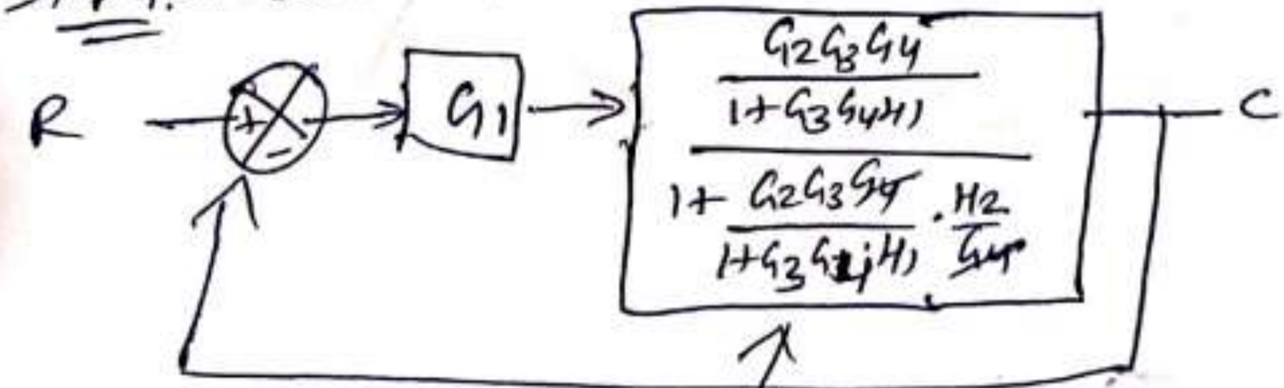
Step 2:- Eliminate Feed Back path & Combine Cascade Blocks



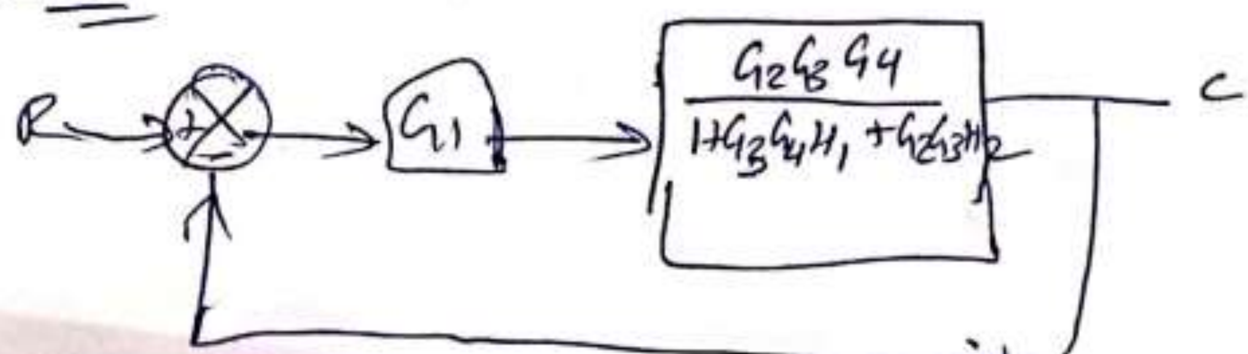
Step 3:- Combine Cascade Blocks.



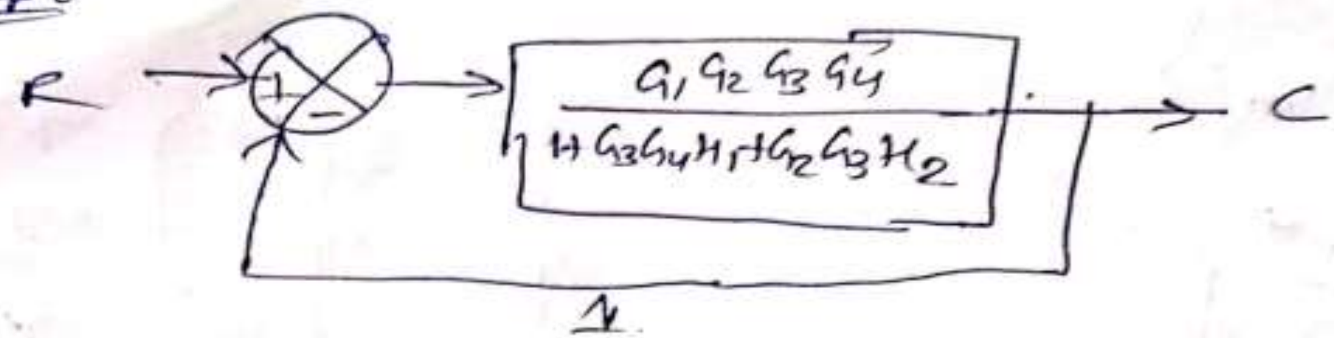
Step 4:- Eliminate Feedback path.



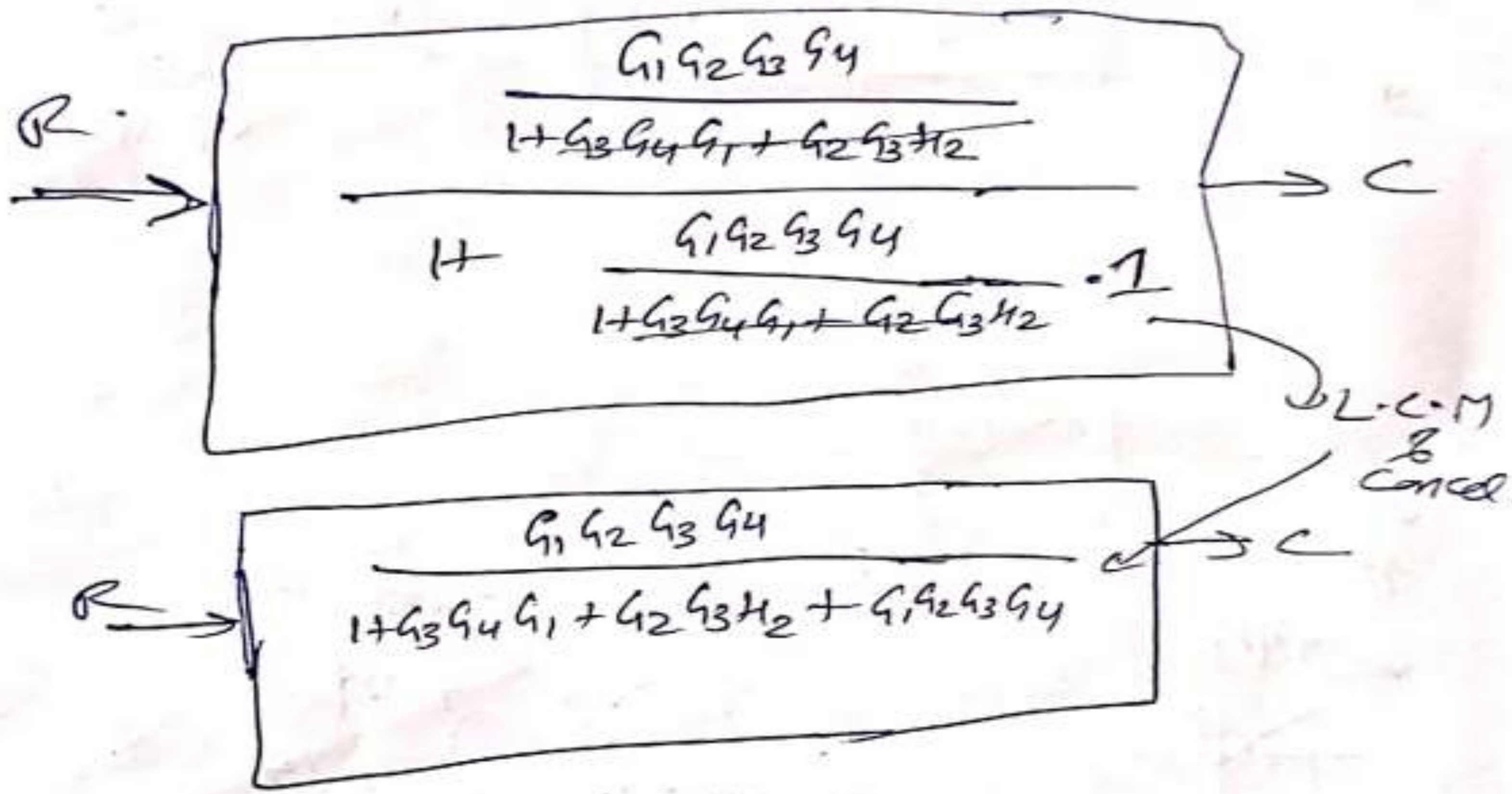
Step 5:- Simplify the term & Combine cascade Blocks



Step 6:- combining cascade blocks



Step 7:- Eliminate feed back loop.



Signal Flow Graph

EXPLANATION OF TERMS USED IN SIGNAL FLOW GRAPH

Node	:	A node is a point representing a variable or signal.
Branch	:	A branch is directed line segment joining two nodes. The arrow on the branch indicates the direction of signal flow and the gain of a branch is the transmittance.
Transmittance	:	The gain acquired by the signal when it travels from one node to another is called transmittance. The transmittance can be real or complex.
Input node (Source)	:	It is a node that has only outgoing branches.
Output node (Sink)	:	It is a node that has only incoming branches.
Mixed node	:	It is a node that has both incoming and outgoing branches.
Path	:	A path is a traversal of connected branches in the direction of the branch arrows. The path should not cross a node more than once.
Open path	:	A open path starts at a node and ends at another node.
Closed path	:	Closed path starts and ends at same node.
Forward path	:	It is a path from an input node to an output node that does not cross any node more than once.
Forward path gain	:	It is the product of the branch transmittances (gains) of a forward path.
Individual loop	:	It is a closed path starting from a node and after passing through a certain part of a graph arrives at same node without crossing any node more than once.
Loop gain	:	It is the product of the branch transmittances (gains) of a loop.
Non-touching Loops	:	If the loops does not have a common node then they are said to be non-touching loops.

Reduction of Signal Flow Graph using MASONS GAIN Formula

$$T = \frac{1}{\Delta} \sum_K P_K \Delta_K$$

T = T(s) = Transfer function of the system

P_K = Forward path gain of K^{th} forward path

K = Number of forward paths in the signal flow graph

Δ = 1 – (Sum of individual loop gains)

$+ \left(\begin{array}{l}\text{Sum of gain products of all possible} \\ \text{combinations of two non - touching loops}\end{array}\right)$

$- \left(\begin{array}{l}\text{Sum of gain products of all possible} \\ \text{combinations of three non - touching loops}\end{array}\right)$

+ ....

Δ_K = Δ for that part of the graph which is not touching K^{th} forward

Problem 1

Find the overall gain C(s)/R(s) for the signal flow graph shown in fig 1.

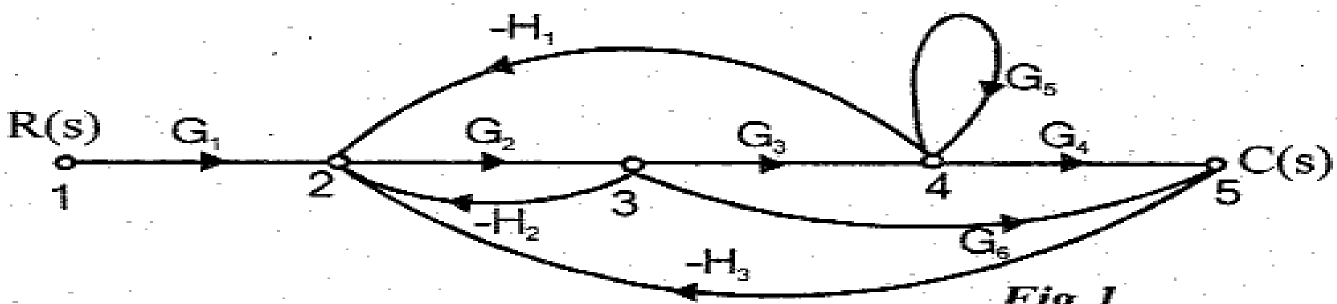


Fig 1

I. Forward Path Gains

There are two forward paths. $\therefore$ K = 2. Let the forward path gains be P_1 and P_2.

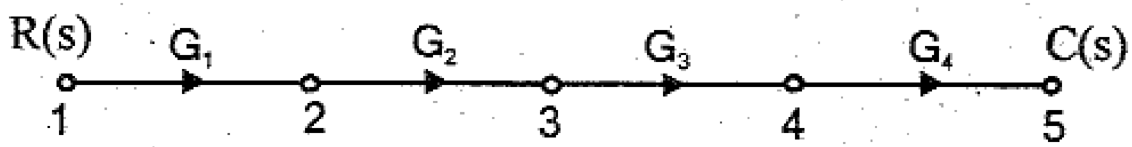


***Fig 2 :** Forward path-1*

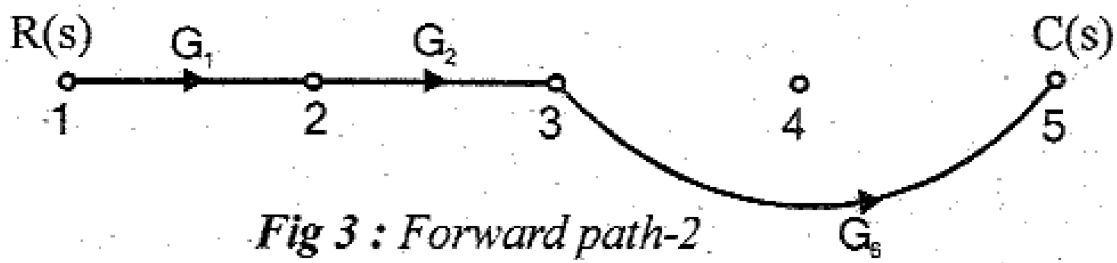


***Fig 3 :** Forward path-2*

Gain of forward path-1, $P_1 = G_1G_2G_3G_4$

Gain of forward path-2, $P_2 = G_1G_2G_6$

Find the overall gain C(s)/R(s) for the signal flow graph shown in fig 1.

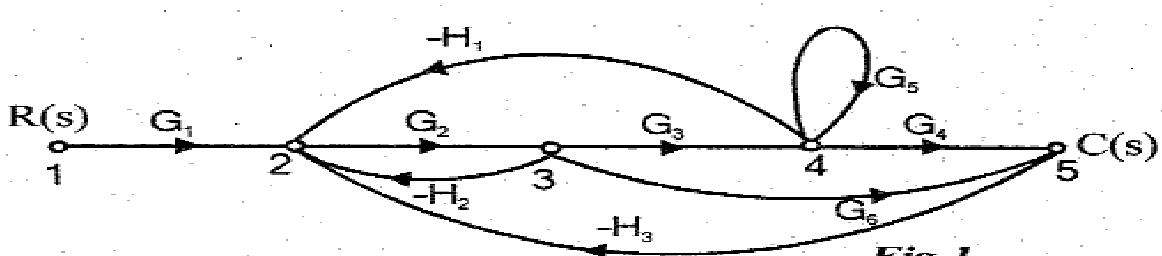


Fig 1

Individual Loop Gain

There are five individual loops. Let the individual loop gains be p_{11}, p_{21}, p_{31}, p_{41} and p_{51}.

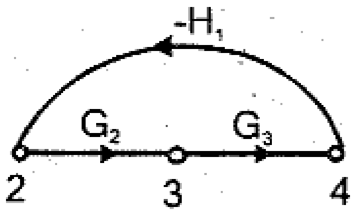


***Fig 4 :** loop-1*

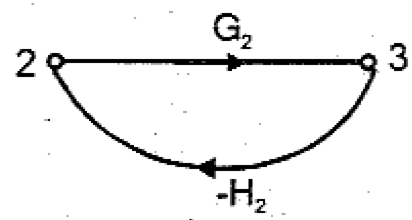


***Fig 5 :** loop-2*

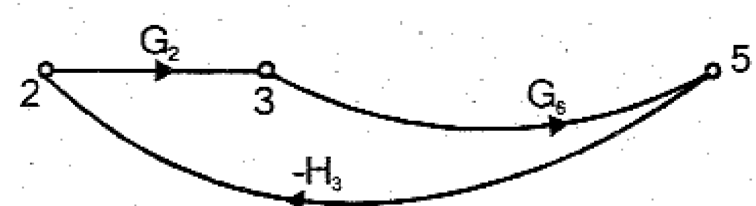


***Fig 6 :** loop-3*

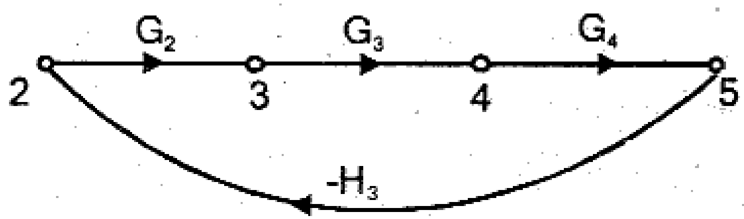


***Fig 7 :** loop-4*

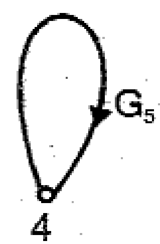


***Fig 8 :** loop-5*

Loop gain of individual loop-1, $P_{11} = -G_2G_3H_1$

Loop gain of individual loop-2, $P_{21} = -H_2G_2$

Loop gain of individual loop-3, $P_{31} = -G_2G_6H_3$

Loop gain of individual loop-4, $P_{41} = -G_2G_3G_4H_3$

Loop gain of individual loop-5, $P_{51} = G_5$

There are five individual loops. Let the individual loop gains be p_{11}, p_{21}, p_{31}, p_{41} and p_{51}.

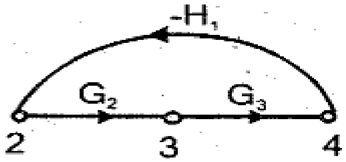


***Fig 4 :** loop-1*

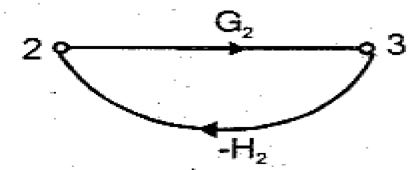


***Fig 5 :** loop-2*

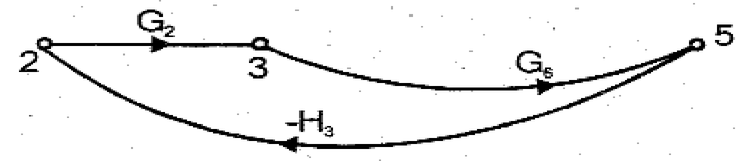


***Fig 6 :** loop-3*

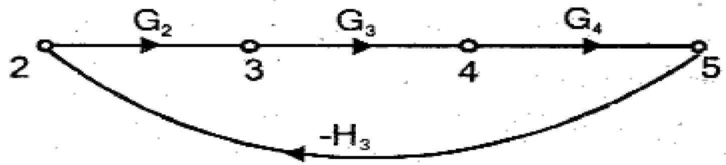


***Fig 7 :** loop-4*

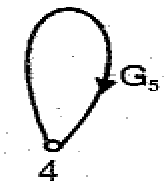


***Fig 8 :** loop-5*

Gain Products of Two Non-touching Loops

There are two combinations of two non-touching loops.

Let the gain products of two non-touching loops be P_{12} and P_{22}.

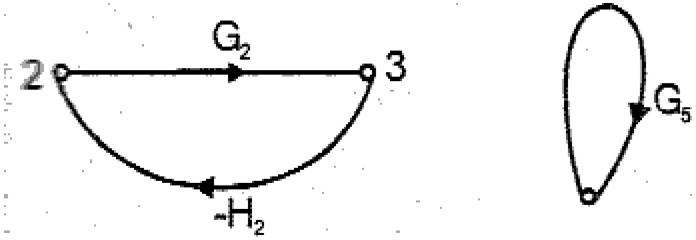


***Fig 9 :** First combination of two non-touching loops*

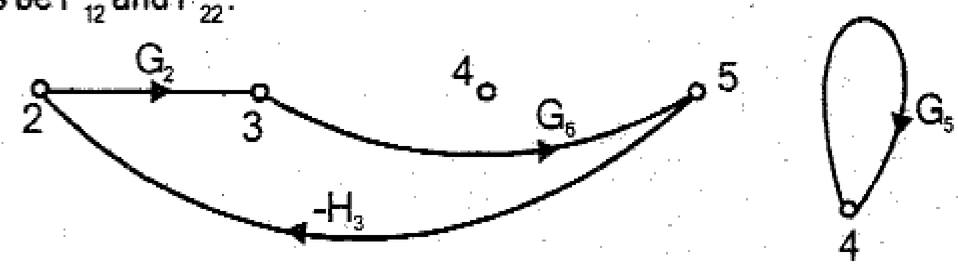


***Fig 10 :** Second combination of two non-touching loops*

Gain product of first combination of two non touching loops $\left.\right\}$ $P_{12} = P_{21}P_{51} = (-G_2H_2)(G_5) = G_2G_5H_2$

Gain product of second combination of two non touching loops $\left.\right\}$ $P_{22} = P_{31}P_{51} = (-G_2G_6H_3)(G_5) = -G_2G_5G_6H_3$

Calculation of Δ and Δ_K

$$\Delta = 1-(P_{11}+P_{21}+P_{31}+P_{41}+P_{51})+(P_{12}+P_{22})$$
$$= 1-(-G_2G_3H_1 - H_2G_2 - G_2G_3G_4H_3 + G_5 - G_2G_6H_3)$$
$$+(-G_2H_2G_5 - G_2G_5G_6H_3)$$

Since there is no part of graph which is not touching forward path-1, $\Delta_1 = 1$.

The part of graph which is not touching forward path-2 is shown in fig 11.

Fig 11

$$\therefore \Delta_2 = 1 - G_5$$

Transfer Function, T

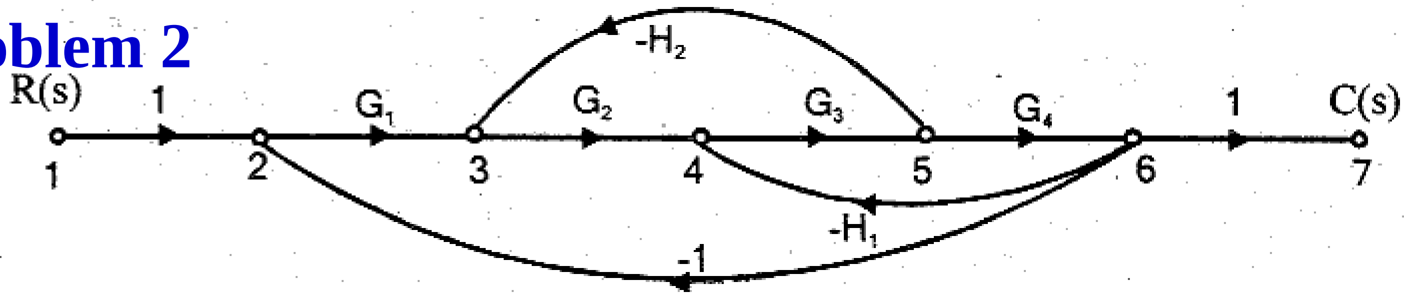
By Mason's gain formula the transfer function, T is given by,

$$T = \frac{1}{\Delta}\sum_K P_K\Delta_K \text{ (Number of forward path is 2 and so K = 2)}$$

$$= \frac{1}{\Delta}\left[P_1\Delta_1 + P_2\Delta_2\right] = \frac{1}{\Delta}\left[G_1G_2G_3G_4 \times 1 + G_1G_2G_6(1-G_5)\right]$$

$$= \frac{G_1G_2G_3G_4 + G_1G_2G_6 - G_1G_2G_5G_6}{1+G_2G_3H_1 + H_2G_2 + G_2G_3G_4H_3 - G_5 + G_2G_6H_3 - G_2H_2G_5 - G_2G_5G_6H_3}$$

Problem 2



I. Forward Path Gains

There is only one forward path. $\therefore K = 1$.

Let the forward path gain be P_1.

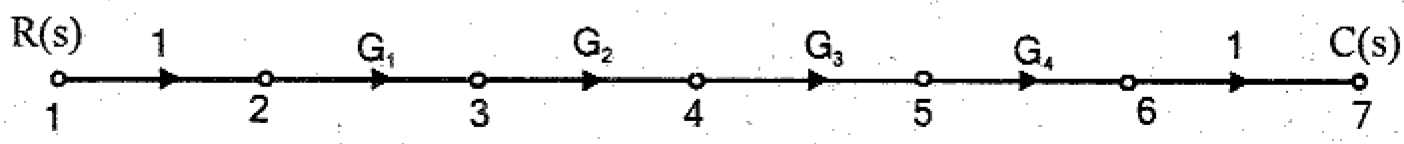


Fig 1 : Forward path-1

Gain of forward path-1, $P_1 = G_1G_2G_3G_4$

II. Individual Loop Gain

There are three individual loops. Let the loop gains be P_{11}, P_{21}, P_{31}.

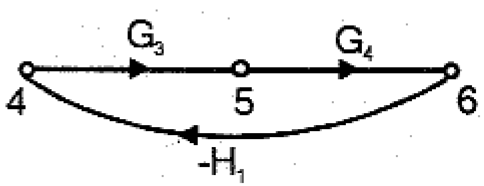


Fig 3 : loop-1

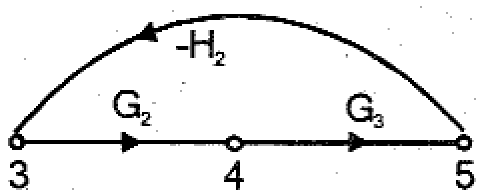


Fig 4 : loop-2

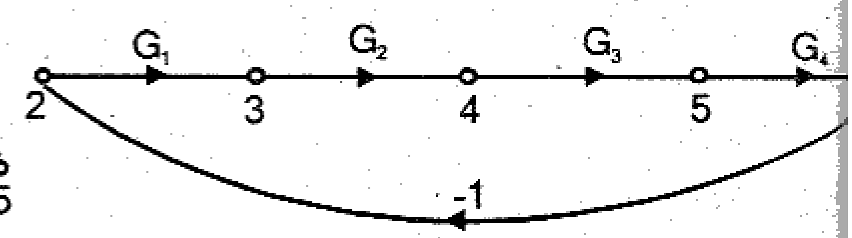


Fig 5 : loop-3

Loop gain of individual loop-1, $P_{11} = -G_3G_4H_1$

Loop gain of individual loop-2, $P_{21} = -G_2G_3H_2$

Loop gain of individual loop-3, $P_{31} = -G_1G_2G_3G_4$

III. Gain Products of Two Non-touching Loops

There are no possible combinations of two non-touching loops, three non-touching loops, etc.

IV. Calculation of Δ and Δ_K

$$\Delta = 1-(P_{11}+P_{21}+P_{31})$$
$$= 1-(-G_3G_4H_1 - G_2G_3H_2 - G_1G_2G_3G_4)$$
$$= 1+G_3G_4H_1 + G_2G_3H_2 + G_1G_2G_3G_4$$

Since no part of the graph is non-touching with forward path-1, $\Delta_1 = 1$.

Transfer Function, T

By Mason's gain formula the transfer function, T is given by,

$$T = \frac{C(s)}{R(s)} = \frac{1}{\Delta}\sum_K P_K\Delta_K = \frac{1}{\Delta}P_1\Delta_1 \text{ (Number of forward path is 1 and so K = 1)}$$

$$= \frac{G_1G_2G_3G_4}{1+G_3G_4H_1 + G_2G_3H_2 + G_1G_2G_3G_4}$$

Problem 3

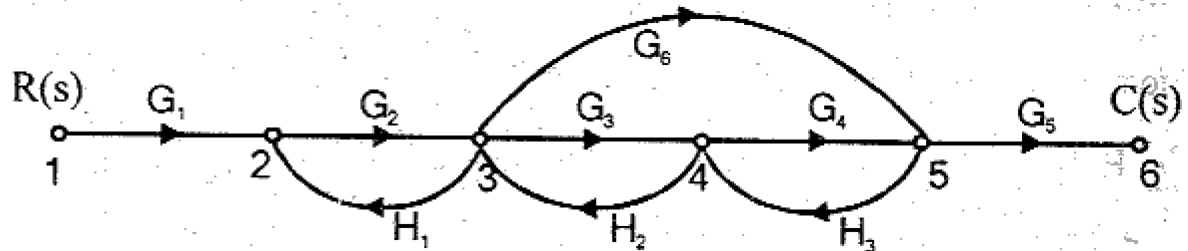


Fig 1

Forward Path Gains

There are two forward paths. $\therefore$ K = 2.

Let forward path gains be P_1 and P_2.

***Fig 2 :** Forward path-1*

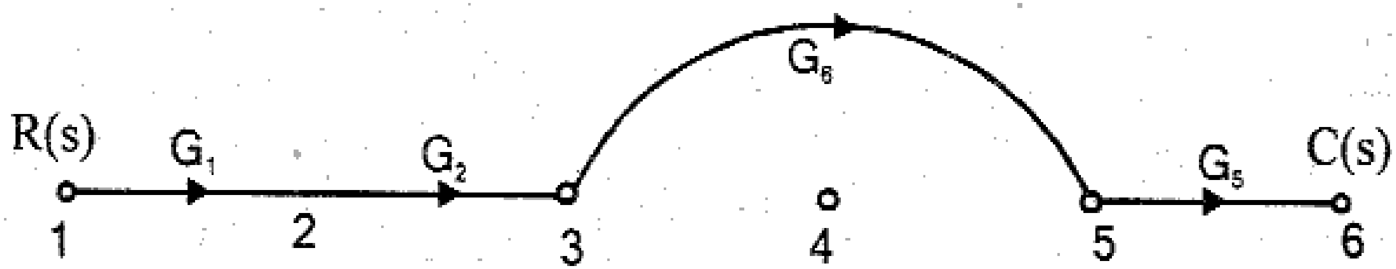


***Fig 3 :** Forward path-2*

Gain of forward path-1, $P_1 = G_1 G_2 G_3 G_4 G_5$

Gain of forward path-2, $P_2 = G_1 G_2 G_6 G_5$

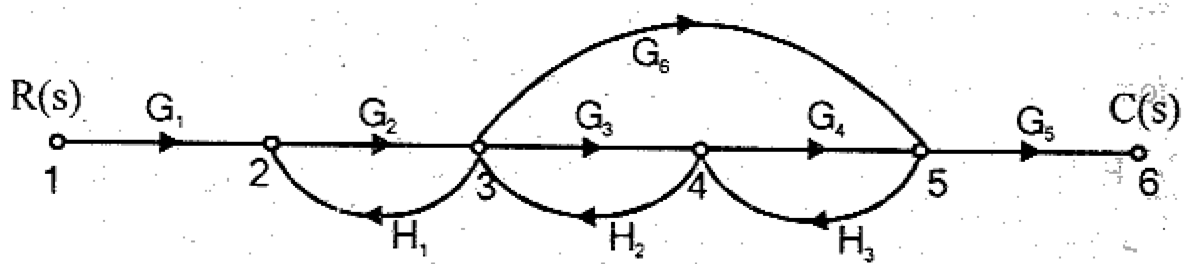


Fig 1

Individual Loop Gain

There are four individual loops. Let individual loop gains be P_{11}, P_{21}, P_{31} and P_{41}.

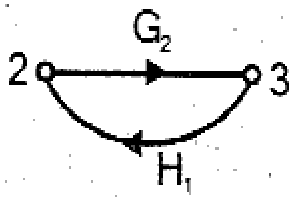


***Fig 4 :** loop-1*

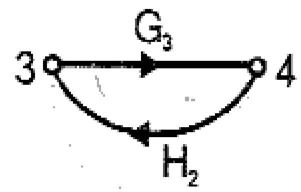


***Fig 5 :** loop-2*

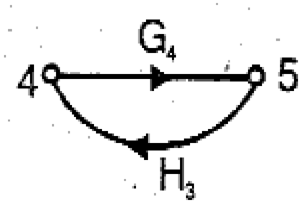


***Fig 6 :** loop-3*

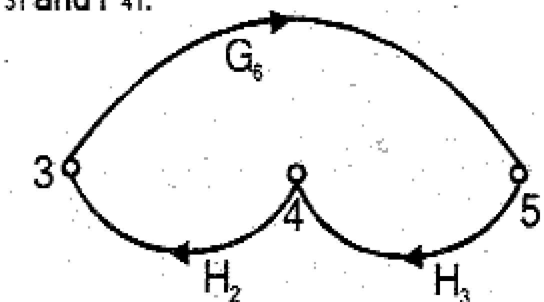


***Fig 7 :** loop-4*

Loop gain of individual loop-1, $P_{11} = G_2H_1$

Loop gain of individual loop-2, $P_{21} = G_3H_2$

Loop gain of individual loop-3, $P_{31} = G_4H_3$

Loop gain of individual loop-4, $P_{41} = G_6H_2H_3$

Gain Products of Two Non-touching Loops

There is only one combination of two non-touching loops. Let the gain ucts of two non-touching loops be P_{12}.

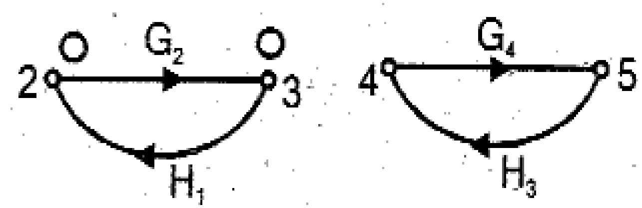


***Fig 8 :** First combination of two non touching loops*

$$\left.\begin{array}{l}\text{Gain product of first combination}\\ \text{of two non - touching loops}\end{array}\right\} P_{12} = (G_2H_1)\,(G_4H_3)$$

$$= G_2G_4H_1H_3$$

IV. Calculation of Δ and Δ_K

$$\Delta = 1-(P_{11}+P_{21}+P_{31}+P_{41})+P_{12}$$
$$= 1-(G_2H_1+G_3H_2+G_4H_3+G_6H_2H_3)+G_2G_4H_1H_3$$
$$= 1-G_2H_1-G_3H_2-G_4H_3-G_6H_2H_3+G_2G_4H_1H_3$$

Since there is no part of graph which is non-touching with forward path-1 and 2, $\Delta_1 = \Delta_2 = 1$

V. Transfer Function, T

By Mason's gain formula the transfer function, T is given by,

$$T = \frac{1}{\Delta}\sum_K P_K\Delta_K = \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2) \quad \text{(Number of forward paths is two and so K = 2)}$$

$$= \frac{G_1G_2G_3G_4G_5 + G_1G_2G_5G_6}{1-G_2H_1-G_3H_2-G_4H_3-G_6H_2H_3+G_2G_4H_1H_3}$$

Problem 4

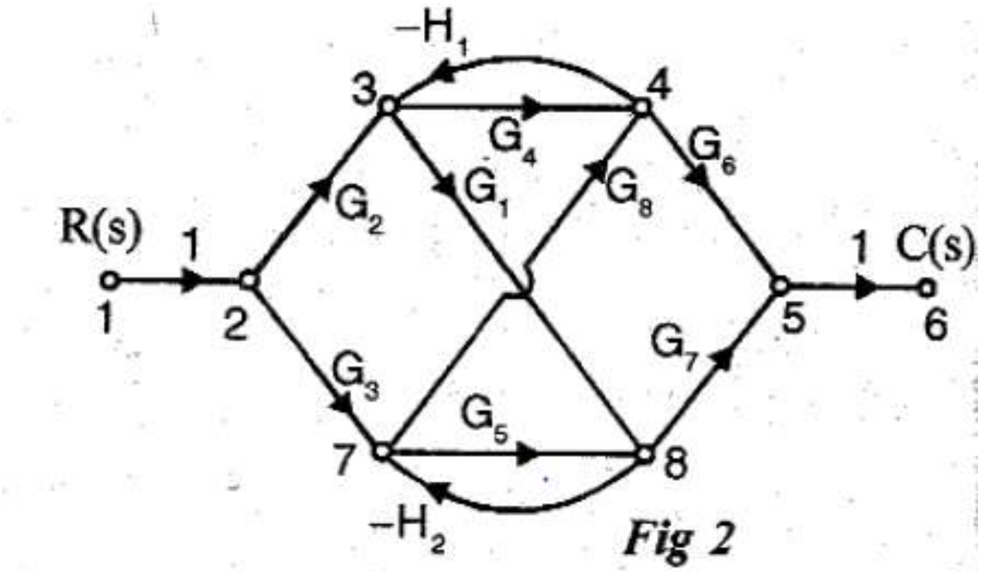


Fig 2

Assume Node Numbers and Start Doing Problem (If not given in Problem)

1. No of Forward Paths- K=6

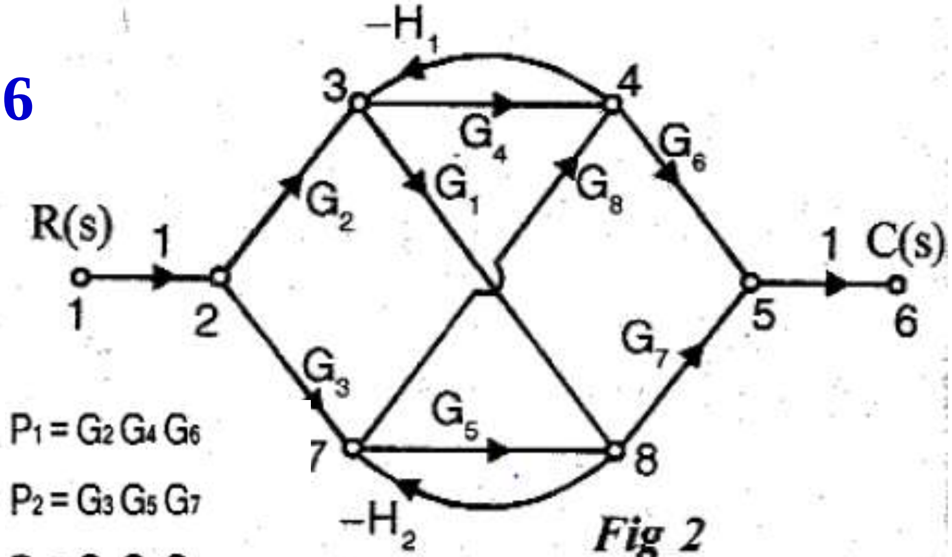
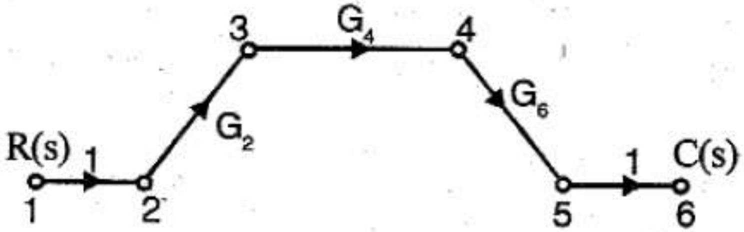
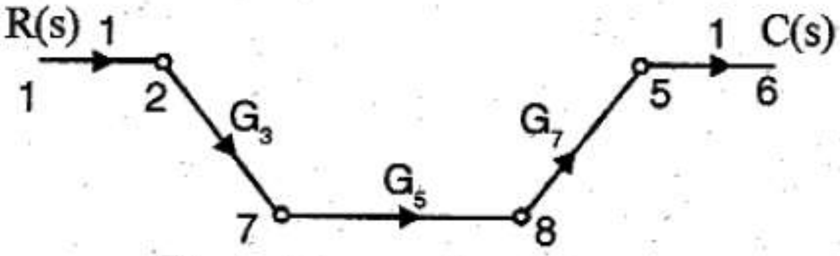


Fig 2





$P_1 = G_2 G_4 G_6$

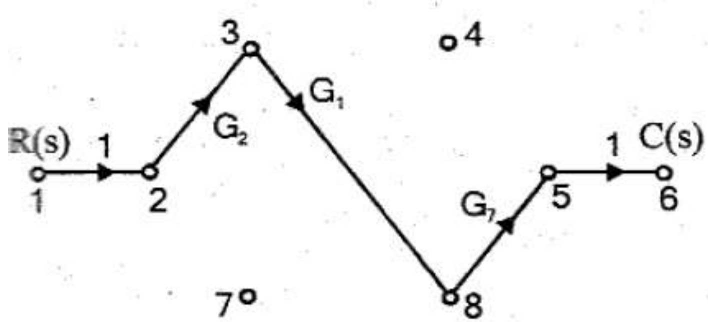
$P_2 = G_3 G_5 G_7$

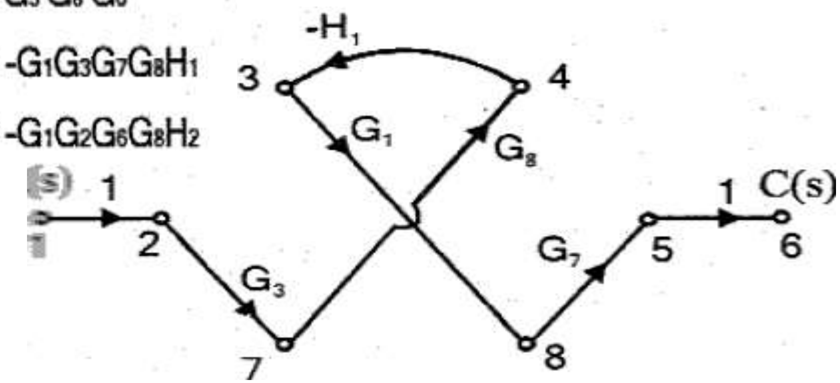
$P_3 = G_1 G_2 G_7$

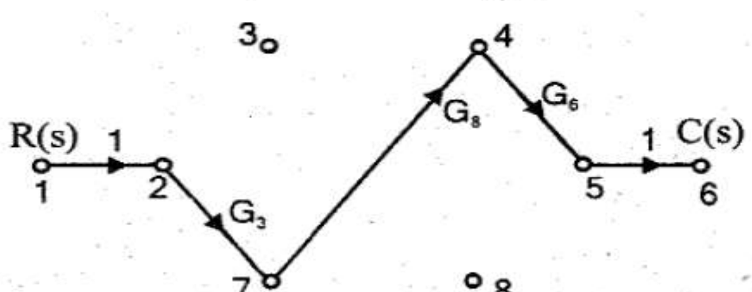
$P_4 = G_3 G_8 G_6$

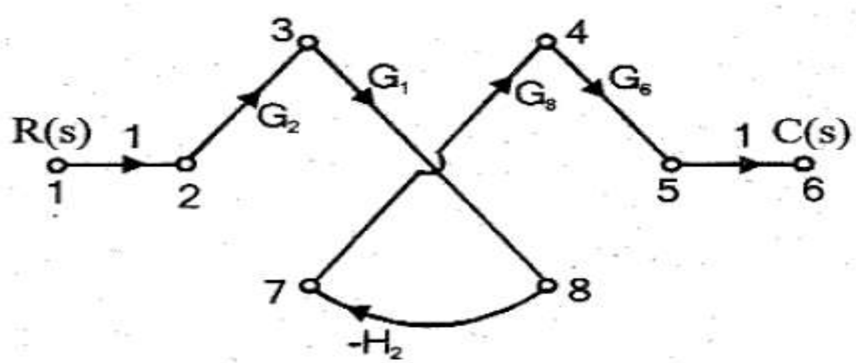
$P_5 = -G_1 G_3 G_7 G_8 H_1$

$P_6 = -G_1 G_2 G_6 G_8 H_2$



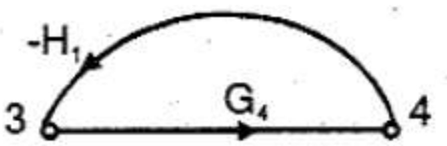


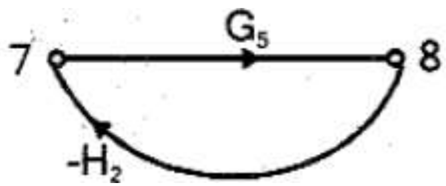




2. Forward Path Gains

3. Individual loops and Gains

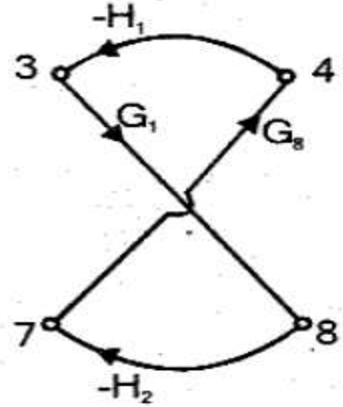




$-G_4H_1$

$-G_5H_2$

$G_1G_8H_1H_2$



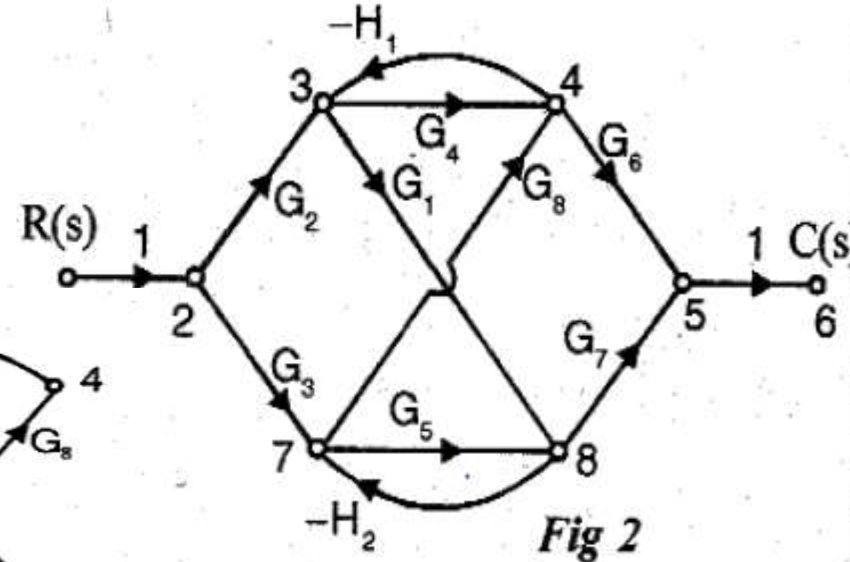
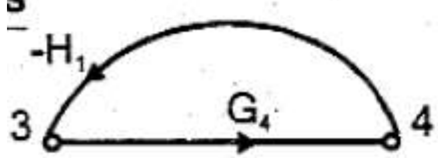
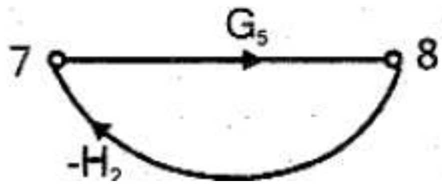


Fig 2

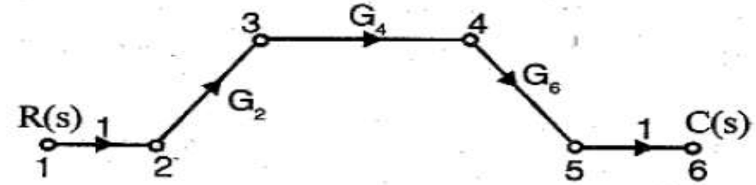
4. Gain Product of two non-touching loops



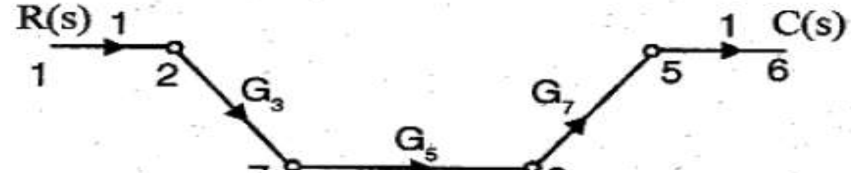


$(-G_4H_1)(-G_5H_2) = G_4G_5H_1H_2$

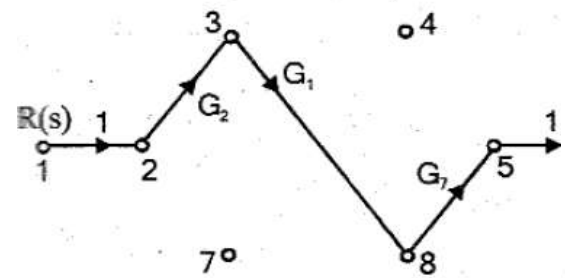
Calculation of Δ and Δ_K

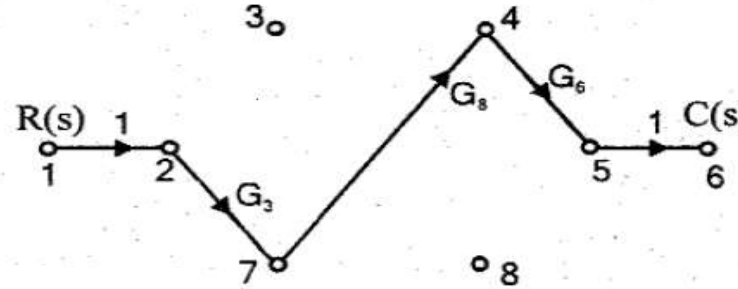


$$\Delta = 1 - (-G_4H_1 - G_5H_2 + G_1G_8H_1H_2) + G_4G_5H_1H_2$$

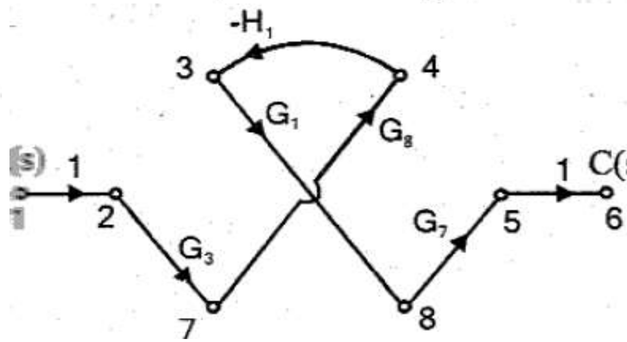


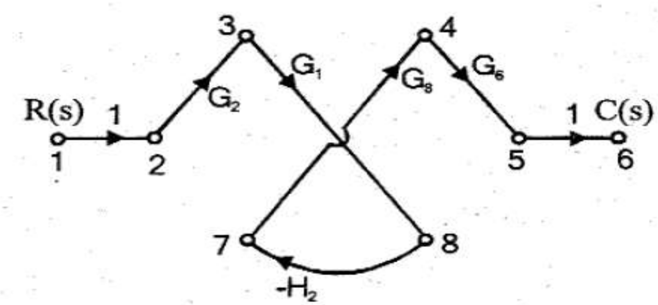
$$\Delta_1 = 1 - (-G_5H_2) = 1 + G_5H_2$$



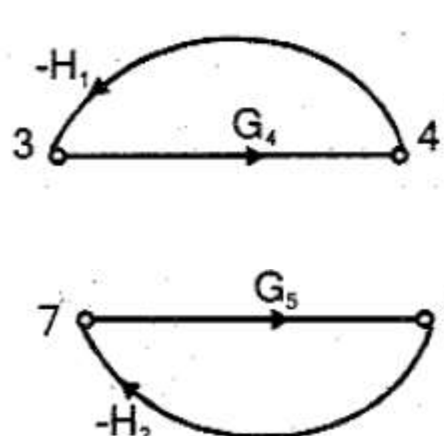


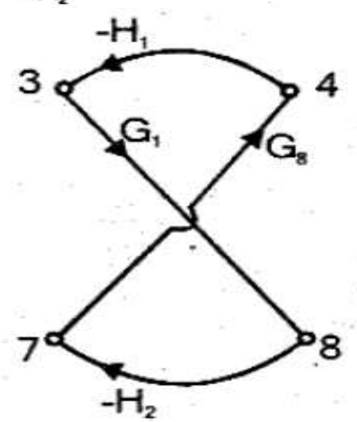
$$\Delta_2 = 1 - (-G_4H_1) = 1 + G_4H_1$$





$$\Delta_3 = \Delta_4 = \Delta_5 = \Delta_6 = 1$$





$$P_1 = G_2 G_4 G_6$$

$$P_2 = G_3 G_5 G_7$$

$$P_3 = G_1 G_2 G_7$$

$$P_4 = G_3 G_8 G_6$$

$$P_5 = -G_1G_3G_7G_8H_1$$

$$P_6 = -G_1G_2G_6G_8H_2$$

$$1 - (-G_4H_1 - G_5H_2 + G_1G_8H_1H_2) + G_4G_5H_1H_2$$

$$\Delta_1 = 1 - (-G_5H_2) = 1 + G_5H_2$$

$$\Delta_2 = 1 - (-G_4H_1) = 1 + G_4H_1$$

$$\Delta_3 = \Delta_4 = \Delta_5 = \Delta_6 = 1$$

By Mason's gain formula the transfer function, T is given by,

$$T = \frac{1}{\Delta}\left(\sum_K P_K \Delta_K\right) \qquad \text{(Number of forward paths is six and so K = 6)}$$

$$= \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4 + P_5\Delta_5 + P_6\Delta_6)$$

$$= \frac{G_2G_4G_6(1+G_5H_2) + G_3G_5G_7(1+G_4H_1) + G_1G_2G_7 + G_3G_6G_8 - G_1G_3G_7G_8H_1 - G_1G_2G_6G_8H_2}{1 + G_4H_1 + G_5H_2 - G_1G_8H_1H_2 + G_4G_5H_1H_2}$$